Change of Electroweak Nuclear Reaction Rates by CP- and Isospin Symmetry Breaking – A Model Calculation

Harald Stumpf

Institute of Theoretical Physics, University Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany

Reprint requests to Prof. H. S.; E-mail: herold@tat.physik.uni-tuebingen.de

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Based on the assumption that electroweak bosons, leptons and quarks possess a substructure of elementary fermionic constituents, in previous papers the effect of CP-symmetry breaking on the effective dynamics of these particles was calculated. Motivated by the phenomenological procedure in this paper, isospin symmetry breaking will be added and the physical consequences of these calculations will be discussed. The dynamical law of the fermionic constituents is given by a relativistically invariant nonlinear spinor field equation with local interaction, canonical quantization, selfregularization and probability interpretation. The corresponding effective dynamics is derived by algebraic weak mapping theorems. In contrast to the commonly applied modifications of the quark mass matrices, CP-symmetry breaking is introduced into this algebraic formalism by an inequivalent vacuum with respect to the CP-invariant case, represented by a modified spinor field propagator. This leads to an extension of the standard model as effective theory which contains besides the "electric" electroweak bosons additional "magnetic" electroweak bosons and corresponding interactions. If furthermore the isospin invariance of the propagator is broken too, it will be demonstrated in detail that in combination with CP-symmetry breaking this induces a considerable modification of electroweak nuclear reaction rates.

Key words: Nuclear Reaction Rates; Electroweak Processes; Symmetry Breaking.

1. Introduction

The question whether electroweak nuclear processes can be influenced by external operation is of great scientific and technical interest. Early experimental attempts to change the decay constants of various members of the radioactive series under various circumstances were unsuccessful. It was therefore concluded that the decay constants of radioactive substances are independent of special preparations [1]. However, these statements must be updated. Apart from variations of the electroweak coupling constants in the high energy range (cf. [2], chapter 27; [3], chapter 17), even in the low energy range changes of the decay rates are possible [1, 4].

In general such decay rates depend on the nuclear and electronic structure of the atoms as well as on the elementary laws of electroweak reactions. Thus change of decay rates means either to try to modify the atoms involved, or the elementary laws of electroweak reactions or both of them. A promising candidate to achieve this is symmetry breaking, which has an effect on atoms as well as on the basic laws.

It is the purpose of this paper to investigate these effects if CP-symmetry breaking is considered in combination with isospin symmetry breaking in a corresponding phenomenological model. The latter model is defined by an effective theory of fermions and bosons which represents an extended electroweak Standard Model, and which can be derived from an underlying microscopic theory.

Based on the assumption that electroweak bosons, leptons and quarks possess a substructure of elementary fermionic constituents, in [5] and [6], it was demonstrated that under CP-symmetry breaking "electric" and "magnetic" electroweak bosons coexist, and that under the influence of this symmetry breaking charged leptons are transmuted into dyons which interact via the electric and magnetic bosons. The same holds for the interplay of leptons and quarks, etc.

The dynamical law for the fermionic constituents of these particles is assumed to be a relativistically invariant nonlinear spinor field with local interaction, canonical quantization, selfregularization and probability interpretation [7]. The corresponding effective theory is derived by means of weak mapping theorems and turns out to be the above mentioned extension of the Standard Model for dyons, where owing to CP-violation the local $SU(2)$ symmetry is simultaneously broken. In this paper we start with this effective theory and explore the consequences for electroweak nuclear reactions.

Concerning the confidence into this procedure it should be noted that this effective theory goes over into a corresponding gauge theory if CP-violation is excluded. On the other hand it must be emphasized that the method of introducing CP-violation in our model is completely different from the corresponding method in the conventional theory. While in the Standard Model the CP-symmetry breaking is formally introduced by quark mass matrices with complex parameters (cf. [2], chapter 26), in our approach this symmetry breaking is effected by an appropriate change of the vacuum which mathematically indicates the transition to a new inequivalent field representation and which physically is a common method successfully applied in solid state physics (cf. [8, 9]).

As a consequence of this difference of the methods, the results differ considerably too. While the formal phenomenological method of the Standard Model is meant to explain the decay of K-mesons, the algebraic method of the model under consideration leads to a completely new formulation and structure of the whole theory due to the new inequivalent vacuum.

In the algebraic treatment the calculations lead to remarkable conclusions which seem to correspond to recent experimental results. Long times the possible modifications, for instance, of nuclear electron capture decay rates were considered as very small. But recently experiments were reported which show that much larger deviations from the common reaction rates can be achieved [10–12]. But the mechanism is unknown, how such results can be obtained. It is the intention of this paper to propose a theoretical reaction scheme which provides a possible basis for the explanation of these experimental results.

As our discussion is based on the results of the preceding papers [5] and [6], it is unavoidable that for brevity we have to refer to these results without giving renewed deductions. In these deductions no use was made of the decomposition into left-handed and right-handed fermions for simplicity. Insofar the model under consideration is a simplified version of the mathematical structure of the Standard Model. This is justified, as already in this version the crucial effects of CP- and isospin symmetry breaking can be demonstrated.

2. Effective Canonical Equations of Motion

The most important theoretical result of the preceding papers [5] and [6] was the derivation of an effective functional energy operator $\mathcal{H}$ which is assumed to represent an extension of the conventional electroweak theory to dyons formulated in functional space. Formally this operator is given by

$$\mathcal{H} = \mathcal{H}_f + \mathcal{H}_b^1 + \mathcal{H}_b^2 + \mathcal{H}_b^3 + \mathcal{H}_{bf}^1 + \mathcal{H}_{bf}^2, \qquad (1)$$

where the various terms of (1) are defined in the order of equation (1) by equations (106), (45), (48), (55), (73) and (104) in [6].

To be in conformity with the phenomenological field definitions of Section 3, it is convenient to carry out a canonical transformation of the functional algebra for the G-fields and E-fields, which is defined by

$$\begin{aligned} b_{la}^G(\mathbf{z}) &= \mathrm{i} b_{la}^G(\mathbf{z})', \quad \partial_{la}^G(\mathbf{z}) = -\mathrm{i}\partial_{la}^G(\mathbf{z})', \\ b_{la}^E(\mathbf{z}) &= -b_{la}^E(\mathbf{z})', \quad \partial_{la}^E(\mathbf{z}) = -\partial_{la}^E(\mathbf{z})', \end{aligned} \qquad (2)$$

while the other algebra elements for the A-fields and the B-fields remain unchanged.

After having performed this transformation in (1) we omit the primes of the new sources in (2) for brevity. With (2) the explicit expressions for the various terms of (1) read

$$\begin{aligned} \mathcal{H}_f = \int & \mathrm{d}^3 z f(\mathbf{z}|B_1 b_1 \alpha_1) \\ & \cdot [-\mathrm{i}(\gamma^0\gamma^k)\partial_k^z + m\gamma^0]_{\alpha_1\alpha_2} \partial^f(\mathbf{z}|B_1 b_1 \alpha_2), \end{aligned} \qquad (3)$$

$$\begin{aligned} \mathcal{H}_b^1 = \mathrm{i} & \int \mathrm{d}^3 z\, b_{la}^A(\mathbf{z})[c_1 \varepsilon_{lkm}\partial_k^z \partial_{ma}^G(\mathbf{z}) - c_2 \partial_{la}^E(\mathbf{z})] \\ & - \mathrm{i} \int \mathrm{d}^3 z\, b_{la}^G(\mathbf{z})[c_1 \varepsilon_{lkm}\partial_k^z \partial_{ma}^A(\mathbf{z}) - c_3 \partial_{la}^B(\mathbf{z})] \\ & + \mathrm{i} \int \mathrm{d}^3 z\, b_{la}^E(\mathbf{z})[\varepsilon_{lkm}\partial_k^z \partial_{ma}^B(\mathbf{z}) + c_2 \partial_{la}^A(\mathbf{z})] \\ & - \mathrm{i} \int \mathrm{d}^3 z\, b_{la}^B(\mathbf{z})[\varepsilon_{lkm}\partial_k^z \partial_{ma}^E(\mathbf{z}) + c_3 \partial_{la}^G(\mathbf{z})], \end{aligned} \qquad (4)$$

$$\begin{aligned} \mathcal{H}_b^2 = -\mathrm{i} & \int \mathrm{d}^3 z \hat{f}^A c_4 b_{la}^E(\mathbf{z}) \partial_{la}^A(\mathbf{z}) \\ & + \mathrm{i} \int \mathrm{d}^3 z \hat{f}^G c_4 b_{la}^B(\mathbf{z}) \partial_{la}^G(\mathbf{z}), \end{aligned} \qquad (5)$$

$$\begin{aligned} \mathcal{H}_b^3 = \bar{\eta}_{abc}\varepsilon_{lkm} \Big\{ 64 \hat{f}^A \int \mathrm{d}^3 z \Big[& k_1 b_{l,a}^A(\mathbf{z}) \partial_{k,b}^A(\mathbf{z}) \partial_{m,c}^G(\mathbf{z}) \\ & + k_2 b_{l,a}^E(\mathbf{z}) \partial_{k,b}^A(\mathbf{z}) \partial_{m,c}^B(\mathbf{z}) \\ & - k_2' b_{l,a}^B(\mathbf{z}) \partial_{k,b}^A(\mathbf{z}) \partial_{m,c}^E(\mathbf{z}) \\ & - k_3 b_{l,a}^G(\mathbf{z}) \partial_{k,b}^A(\mathbf{z}) \partial_{m,c}^A(\mathbf{z}) \Big] \end{aligned}$$

$$+\,64\hat f^G\int \mathrm{d}^3z\,\Big[k_4 b^A_{l,a}(\mathbf{z})\partial^G_{k,b}(\mathbf{z})\partial^A_{m,c}(\mathbf{z}) + k_5 b^E_{l,a}(\mathbf{z})\partial^G_{k,b}(\mathbf{z})\partial^E_{m,c}(\mathbf{z}) + k_5' b^B_{l,a}(\mathbf{z})\partial^G_{k,b}(\mathbf{z})\partial^B_{m,c}(\mathbf{z}) + k_6 b^G_{l,a}(\mathbf{z})\partial^G_{k,b}(\mathbf{z})\partial^G_{m,c}(\mathbf{z})\Big]\Big\}, \tag{6}$$

$$\mathcal{H}^1_{bf} = -K_1\int \mathrm{d}^3z(\gamma^0\gamma^k)_{nm}(T^0\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial^A_{k0}(\mathbf{z})\partial^f_{mj}(\mathbf{z}) + \mathrm{i}K_1\int \mathrm{d}^3z(\gamma^0\gamma^k\gamma^5)_{nm}(S^0\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial^G_{k0}(\mathbf{z})\partial^f_{mj}(\mathbf{z}) + \frac{1}{3}K_1\sum_{b=1}^{3}\int \mathrm{d}^3z(\gamma^0\gamma^k)_{nm}(T^b\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial^A_{kb}(\mathbf{z})\partial^f_{mj}(\mathbf{z}) - \mathrm{i}\frac{1}{3}K_1\sum_{b=1}^{3}\int \mathrm{d}^3z(\gamma^0\gamma^k\gamma^5)_{nm}(S^b\gamma^5)_{lj}f_{nl}(\mathbf{z})\partial^G_{kb}(\mathbf{z})\partial^f_{mj}(\mathbf{z}), \tag{7}$$

$$\mathcal{H}^2_{bf} = \mathrm{i}Kt(0)^4\int \mathrm{d}^3z\,\Theta^n_{B_1b_1,B_2b_2}\cdot\big[-2\,f^E(\gamma^kC)^+_{\mu_1\mu_2}b^E(\mathbf{z}|n,k) + f^B\mathrm{i}(\gamma^5\gamma^kC)^+_{\mu_1\mu_2}b^B(\mathbf{z}|n,k)\big]\cdot\partial^f(\mathbf{z}|B_1,b_1,\mu_1)\partial^f(\mathbf{z}|B_2,b_2,\mu_2). \tag{8}$$

It should be emphasized that the input of equations (3) – (8) is solely the spinor field model ([6], section 2), and its sets of single bosonic and single fermionic bound states ([6], section 4).

A physical interpretation of the associated effective functional energy equation (21) of [6] can be achieved by considering the classical limit of this equation. In this classical limit the system is described by its classical equations of motion. These equations of motion can be exactly derived from equation (21) of [6], if correlations in the matrix elements are suppressed. For details of the corresponding deduction we refer to [13], section 7.5 for instance.

In the field part of this set of equations the quantities E_{la} and B_{la}, $l = 1, 2, 3$ and $a = 0, 1, 2, 3$, represent the $SU(2)\otimes U(1)$ field strengths, while A_{la} and G_{la} are the "electric" and "magnetic" vector potentials in *temporal gauge*. This "gauge" can be selfconsistently justified as a general constraint, even if the original $SU(2)$ invariance is broken. Such vector potentials were introduced by Cabbibo and Ferrari [14] in electrodynamics, and the following set of equations represents an electroweak generalization of this approach:

$$\mathrm{i}\dot A_{la}(\mathbf{z}) = \mathrm{i}c_1\varepsilon_{lkm}\partial^z_k G_{ma}(\mathbf{z}) - \mathrm{i}c_2E_{la}(\mathbf{z}) + \bar\eta_{abc}\varepsilon_{lkm}\big[\hat f^A k_1A_{kb}(\mathbf{z})G_{mc}(\mathbf{z}) + \hat f^G k_4G_{kb}(\mathbf{z})A_{mc}(\mathbf{z})\big], \tag{9}$$

$$\mathrm{i}\dot G_{la}(\mathbf{z}) = -\mathrm{i}c_1\varepsilon_{lkm}\partial^z_k A_{ma}(\mathbf{z}) + \mathrm{i}c_3B_{la}(\mathbf{z}) + \bar\eta_{abc}\varepsilon_{lkm}\big[-\hat f^A k_3A_{kb}(\mathbf{z})A_{mc}(\mathbf{z}) + \hat f^G k_6G_{kb}(\mathbf{z})G_{mc}(\mathbf{z})\big], \tag{10}$$

$$\mathrm{i}\dot E_{la}(\mathbf{z}) = \mathrm{i}\varepsilon_{lkm}\partial^z_k B_{ma}(\mathbf{z}) + \mathrm{i}(c_2-\hat f^A c_4)A_{la}(\mathbf{z}) + \bar\eta_{abc}\varepsilon_{lkm}\big[\hat f^A k_2A_{kb}(\mathbf{z})B_{mc}(\mathbf{z}) + \hat f^G k_5G_{kb}(\mathbf{z})E_{mc}(\mathbf{z})\big] - \mathrm{i}K'\Theta^a_{B_1b_1,B_2b_2}\hat f^E(\gamma^lC)^+_{\mu_1\mu_2}\cdot\psi_{B_1b_1\mu_1}(\mathbf{z})\psi_{B_2b_2\mu_2}(\mathbf{z}), \tag{11}$$

$$\mathrm{i}\dot B_{la}(\mathbf{z}) = -\mathrm{i}\varepsilon_{lkm}\partial^z_k E_{ma}(\mathbf{z}) - \mathrm{i}(c_3-\hat f^G c_4)G_{la}(\mathbf{z}) + \bar\eta_{abc}\varepsilon_{lkm}\big[-\hat f^A k_2'A_{kb}(\mathbf{z})E_{mc}(\mathbf{z}) + \hat f^G k_5'G_{kb}(\mathbf{z})B_{mc}(\mathbf{z})\big] + \mathrm{i}(K'/2)\Theta^a_{B_1b_1,B_2b_2}\hat f^B(\mathrm{i}\gamma^5\gamma^lC)^+_{\mu_1\mu_2}\cdot\psi_{B_1b_1\mu_1}(\mathbf{z})\psi_{B_2b_2\mu_2}(\mathbf{z}). \tag{12}$$

The factor 64 in (6) has been included in the definition of the constants k_i in (9) – (12). For the fermion fields the following equations of motion can be derived:

$$\mathrm{i}\dot\psi_{\alpha l}(\mathbf{z}) = \big[-\mathrm{i}(\gamma^0\gamma^k)_{\alpha\beta}\partial^z_k + \gamma^0_{\alpha\beta}m\big]\psi_{\beta l}(\mathbf{z}) - K_1\big[(\gamma^0\gamma^k)_{\alpha\beta}(T^0\gamma^5)_{ln}A_{k0}(\mathbf{z}) - \mathrm{i}(\gamma^0\gamma^k\gamma^5)_{\alpha\beta}(S^0\gamma^5)_{ln}G_{k0}(\mathbf{z})\big]\psi_{\beta n}(\mathbf{z}) + \frac{1}{3}K_1\sum_{b=1}^{3}\big[(\gamma^0\gamma^k)_{\alpha\beta}(T^b\gamma^5)_{ln}A_{kb}(\mathbf{z}) - \mathrm{i}(\gamma^0\gamma^k\gamma^5)_{\alpha\beta}(S^b\gamma^5)_{ln}G_{kb}(\mathbf{z})\big]\psi_{\beta n}(\mathbf{z}), \tag{13}$$

where the indices l, n refer to the phenomenological numeration of the lepton states. This means that the field quantities $\psi_{\alpha,l}$ are superspinors of the phenomenological theory and ought not to be confused with the spinor field operators of the basic spinor field model in the background. The sets of antisymmetric and symmetric matrices T^a, S^a, $a = 0, 1, 2, 3$, are representatives of the underlying $SU(2)\otimes U(1)$ group structure. They are given by equations (25) and (26) in [6] and read

$$S^l = \begin{pmatrix} 0 & \sigma^l \\ (-1)^{l+1}\sigma^l & 0 \end{pmatrix}, \quad T^l = \begin{pmatrix} 0 & \sigma^l \\ (-1)^l\sigma^l & 0 \end{pmatrix} \tag{14}$$

for the triplet, and

$$S^0 = \begin{pmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{pmatrix}, \quad T^0 = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix} \tag{15}$$

for the singlet.

In (11) and (12) the four-dimensional index κ is splitted into the double index $\kappa = (B,b)$. Formally we define (B,b) by superspinors in the S-representation:

$$\psi^{\mathrm{S}}_{Bb\alpha i}(x) = \begin{pmatrix} \psi_{b\alpha i}(x)\ ;\ B=1 \\ \psi^{c}_{b\alpha i}(x)\ ;\ B=2 \end{pmatrix}. \tag{16}$$

But for technical reasons of the calculation in [6], aside from charge-conjugated spinors in (16) also G-conjugated spinors were introduced, and this definition is also applied in the phenomenological theory. The introduction of G-conjugated spinors allows product representations in superspin-isospin space and is indicated by the superscript D (decomposition). For instance the central formula (92) in [6] of the superspin-isospin part of the current calculation is formulated in D-representation.

To calculate this D-representation we start with the S-representation. According to the construction of formula (92) in [6] the tensor Θ^n on the left hand side of (92) is the superspin-isospin part of the boson dual function $R^k_{q_1q_2}$ of (74) in [6]. Thus we start first with the superspin-isospin parts of the original boson functions $C^k_{q_1q_2}$, construct their duals and transform these duals from the S- into the D-representation. The original superspin-isospin basis set of the $C^k_{q_1q_2}$ functions reads for the case of CP-symmetry breaking (cf. (27) in [6])

$$\begin{aligned}(\Theta^a)^{\mathrm{S}}_{\kappa_1\kappa_2} &:= \frac{1}{2}(T^a + S^a)^{\mathrm{S}}_{\kappa_1\kappa_2} \\ &:= \frac{1}{2}(\mathrm{i}\sigma^2 + \sigma^1)_{B_1B_2} \otimes \sigma^a_{b_1b_2}, \\ a &= 0,1,2,3.\end{aligned} \tag{17}$$

Its dual set $\tilde{\Theta}^n$, $n = 0, 1, 2, 3$, is given by

$$\begin{aligned}(\tilde{\Theta}^n)^{\mathrm{S}}_{\kappa_1\kappa_2} &= (\tilde{\Theta}^n)^{\mathrm{S}}_{B_1b_1B_2b_2} \\ &= \frac{1}{2}(\mathrm{i}\sigma^2 + \sigma^1)_{B_1B_2}(\sigma^n)^T_{b_1b_2}.\end{aligned} \tag{18}$$

Owing to the properties of the Pauli algebra one easily verifies that the duality relations

$$\begin{aligned}&(\tilde{\Theta}^n)^{\mathrm{S}}_{\kappa_1\kappa_2}(\Theta^{n'})^{\mathrm{S}}_{\kappa_1\kappa_2} \\ &= \frac{1}{4}(\mathrm{i}\sigma^2 + \sigma^1)_{B_1B_2}(\mathrm{i}\sigma^2 + \sigma^1)_{B_1B_2}(\sigma^n)^T_{b_1b_2}\sigma^{n'}_{b_1b_2} \\ &= 2\delta_{nn'}\end{aligned} \tag{19}$$

are satisfied. In (19) the state normalization is omitted because it is irrelevant, see below. In the next step we transform the tensor (18) from the S- into the D-representation.

The transformation law of the superspin-isospin part (17) of the boson functions $C^k_{q_1q_2}$ is defined by the relation

$$(\Theta^n)^{\mathrm{S}}_{\kappa_1\kappa_2} = G_{\kappa_1\kappa'_1}G_{\kappa_2\kappa'_2}(\Theta^n)^{\mathrm{D}}_{\kappa'_1\kappa'_2} \tag{20}$$

with the transformation matrix

$$G := \begin{pmatrix} \mathbf{1} & 0 \\ 0 & -\mathrm{i}\sigma_2 \end{pmatrix}. \tag{21}$$

The duality relation (19) has to be invariant under the change of the representation. This means that

$$(\tilde{\Theta}^n)^{\mathrm{S}}_{\kappa_1\kappa_2}(\Theta^{n'})^{\mathrm{S}}_{\kappa_1\kappa_2} = (\tilde{\Theta}^n)^{\mathrm{D}}_{\kappa_1\kappa_2}(\Theta^{n'})^{\mathrm{D}}_{\kappa_1\kappa_2} \tag{22}$$

has to hold, which leads to the transformation law for the dual set

$$(\tilde{\Theta}^n)^{\mathrm{S}}_{\kappa_1\kappa_2} = G^{-1}_{\kappa_1\kappa'_2}G^{-1}_{\kappa_2\kappa'_2}(\tilde{\Theta}^n)^{\mathrm{D}}_{\kappa'_1\kappa'_2} \tag{23}$$

or

$$(\tilde{\Theta}^n)^{\mathrm{D}}_{\kappa_1\kappa_2} = G_{\kappa_1\kappa'_2}G_{\kappa_2\kappa'_2}(\tilde{\Theta}^n)^{\mathrm{S}}_{\kappa'_1\kappa'_2}. \tag{24}$$

With (18), (21), (24), and $c = -\mathrm{i}\sigma_2$ one obtains

$$\begin{aligned}(\tilde{\Theta}^n)^{\mathrm{D}}_{\kappa_1\kappa_2} &= \frac{1}{2}(\mathrm{i}\sigma^2 + \sigma^1)_{B_1B_2}[(\sigma^n)^T c^T]_{b_1b_2} \\ &\equiv \delta_{B_11}\delta_{B_22}[(\sigma^n)^T c^T]_{b_1b_2}.\end{aligned} \tag{25}$$

In consequence of (25) equation (92) of [6] must be corrected by replacing $(\Theta^n)^{\mathrm{D}}$ by $(\tilde{\Theta}^n)^{\mathrm{D}}$. This yields the revised formula

$$\begin{aligned}S^{A_1a_1A_2a_2}_{B_1b_1B_2b_2}(\tilde{\Theta}^n)^{\mathrm{D}}_{[A_1]a_1[A_2]a_2} &= (\bar{\Theta}^n)^{\mathrm{D}}_{B_1b_1B_2b_2} \\ &:= (\tilde{\Theta}^n)^{\mathrm{D}}_{B_2b_1B_1b_2},\end{aligned} \tag{26}$$

where $\bar{\Theta}^n$ is an auxiliary tensor defined by $\tilde{\Theta}^n$ on the right hand side of (26). Therefore in [6] all following equations have to be corrected in accordance with this correction. This includes the correction of the current expressions in (11) and (12) which correspond to the currents in (114) and (115) in [6]. For instance, in (11) the electric current has to be replaced by

$$j^a_l = (\bar{\Theta}^a_{B_1b_1B_2b_2})^{\mathrm{D}}(\gamma^l C)^+_{\mu_1\mu_2}\psi_{B_1b_1\mu_1}\psi_{B_2b_2\mu_2}. \tag{27}$$

To evaluate this expression, the definition of the phenomenological spinor fields has to be given. According to the construction of lepton states, in this case the numbers (B,b) are referred to superspinors in D-representation. Owing to (37) and (85) in [6] one obtains

$$\begin{aligned}\psi^{\mathrm{D}}_{1,1,\mu} &\equiv e^{+}_{\mu}, \quad \psi^{\mathrm{D}}_{1,2,\mu} \equiv \bar{\nu}_{\mu},\\ \psi^{\mathrm{D}}_{2,1,\mu} &\equiv \nu_{\mu}, \quad \psi^{\mathrm{D}}_{2,2,\mu} \equiv e^{-}_{\mu}.\end{aligned} \tag{28}$$

Then, with (25) and the definition (26), (27) reads

$$\begin{aligned} j^a_l &= \frac{1}{2}\delta_{2B_1}\delta_{1B_2}[(\sigma^a)^T c^T]_{b_1b_2}(\gamma^l C)^{+}_{\mu_1\mu_2}\psi^{\mathrm{D}}_{B_1b_1\mu_1}\psi^{\mathrm{D}}_{B_2b_2\mu_2}\\ &= \frac{1}{2}[(\sigma^a)^T c^T]_{b_1b_2}(\gamma^l C)^{+}_{\mu_1\mu_2}\psi^{\mathrm{D}}_{2,b_1\mu_1}\psi^{\mathrm{D}}_{1,b_2\mu_2}. \end{aligned} \tag{29}$$

The phenomenological fields in S-representation are defined by $\psi^{\mathrm{D}}_{2,1,\mu} \equiv \psi^{\mathrm{S}}_{2,1,\mu} \equiv \nu_\mu$ and $\psi^{\mathrm{D}}_{2,2,\mu} \equiv \psi^{\mathrm{S}}_{2,2,\mu} \equiv e^{-}_{\mu}$ and their charge conjugated counterparts. The latter can be generated by the transformation $\psi^{\mathrm{D}}_{1,b,\mu} = c^T_{b,b'}\psi^{\mathrm{S}}_{1,b',\mu}$. Therefore (29) can be rewritten into the form

$$\begin{aligned} j^a_l &= \frac{1}{2}[(\sigma^a)^T c^T c^T]_{b_1b_2}(\gamma^l C)^{+}_{\mu_1\mu_2}\psi^{\mathrm{S}}_{2,b_1,\mu_1}\psi^{\mathrm{S}}_{1,b_2\mu_2}\\ &= -\frac{1}{2}(\sigma^a)^T_{b_1b_2}(\gamma^l C)^{+}_{\mu_1\mu_2}\psi_{b_1\mu_1}\psi^{c}_{b_2\mu_2}. \end{aligned} \tag{30}$$

With $(\gamma^l C)$ its Hermitean conjugate is symmetric too. Thus (29) reads equivalently

$$j^a_l = -(\psi^c_{b_2\mu_2})^T(\sigma^a)_{b_2b_1}(\gamma^l C)^{+}_{\mu_2\mu_1}\psi_{b_1\mu_1}. \tag{31}$$

In the last step one uses $(\psi^c)^T = \bar{\psi}C^T$ and obtains from (31) the $U(1)$ and $SU(2)$ currents

$$j^a_l \equiv -\frac{1}{2}\bar{\psi}_{b_1\mu_1}\sigma^a_{b_1b_2}\gamma^l_{\mu_1\mu_2}\psi_{b_2\mu_2}. \tag{32}$$

In the same way one can proceed to get the magnetic currents J^a_l. The factors (1/2) will be absorbed in the coupling constants, i. e., normalization of the states is irrelevant.

In the next step we rearrange the Dirac equation (13) into the conventional form. For the interpretation of (13) it is important to realize that the $(T\gamma^5)$ and $(S\gamma^5)$ matrices in (13) arise from matrix elements between two three-parton states which characterize the superspin-isospin part of the composite leptons (see [6], (68),(69)). As the lepton states are constructed in a D-basis of parton spinors the latter matrix elements have to be calculated in this basis. The calculation yields for $a = 1, 2, 3$

$$(S^a\gamma^5)^{\mathrm{D}}_{ln} = \begin{pmatrix}\sigma^a & 0\\ 0 & -\sigma^a\end{pmatrix}, \quad (T^a\gamma^5)^{\mathrm{D}}_{ln} = \begin{pmatrix}\sigma^a & 0\\ 0 & \sigma^a\end{pmatrix}, \tag{33}$$

and for $a = 0$

$$(S^0\gamma^5)^{\mathrm{D}}_{ln} = \begin{pmatrix}\mathbf{1} & 0\\ 0 & \mathbf{1}\end{pmatrix}, \qquad (T^0\gamma^5)^{\mathrm{D}}_{ln} = \begin{pmatrix}\mathbf{1} & 0\\ 0 & -\mathbf{1}\end{pmatrix}, \tag{34}$$

where the indices l, n are referred to the state numbers of (28).

Substitution of (33) and (34) into (13) shows that this equation can be decomposed into two separate equations for ψ_1, ψ_2 and ψ_3, ψ_4. In particular, for $(\psi_3, \psi_4) \equiv (\nu, e^-)$ one obtains after multiplication of (13) with γ^0 in spin-space the equation

$$\begin{aligned} &[-\mathrm{i}\gamma^\mu\partial_\mu + m]\psi_l + \frac{1}{2}[g\sigma^a_{ln}\gamma^k A_{ka} + g'\sigma^0_{ln}\gamma^k A_{k0}]\psi_n\\ &+\mathrm{i}\frac{1}{2}[g\sigma^a_{ln}(\gamma^k\gamma^5)G_{ka} + g'\sigma^0_{ln}(\gamma^k\gamma^5)G_{k0}]\psi_n = 0. \end{aligned} \tag{35}$$

The corresponding equation for (ψ_1, ψ_2) is redundant and will not be explicitly given for the sake of brevity.

Finally we rearrange the field equations into their final form. Neglecting for simplicity the coupling between $SU(2)$ fields and $U(1)$ fields from (53) in [6] it follows $\bar{\eta}_{abc} := \mathrm{i}\varepsilon_{abc}$. Furthermore we define $c_1 = 1$, $\hat{f}^A = \hat{f}^G$, $k_2 = k'_2$, $k_5 = k'_5$, $(c_2 - \hat{f}^A c_4) =: \mu_A$ and $(c_3 - \hat{f}^A c_4) =: \mu_G$, and express the current coupling constants g_e and g_m by the original constants in (11) and (12).

Substitution of these definitions and canceling out i yields for (9)–(12) the following set of field equations:

$$\begin{aligned} \dot{A}_{la}(\mathbf{z}) &= -\varepsilon_{lkm}\partial^z_k G_{ma}(\mathbf{z}) - c_2 E_{la}(\mathbf{z})\\ &+\varepsilon_{abc}\varepsilon_{lkm}\hat{f}^A[k_1 A_{kb}(\mathbf{z})G'_{mc}(\mathbf{z}) + k_4 G_{kb}(\mathbf{z})A_{mc}(\mathbf{z})], \end{aligned} \tag{36}$$

$$\begin{aligned} \dot{G}_{la}(\mathbf{z}) &= -\varepsilon_{lkm}\partial^z_k A_{ma}(\mathbf{z}) + c_2 B_{la}(\mathbf{z})\\ &-\varepsilon_{abc}\varepsilon_{lkm}\hat{f}^A[k_3 A_{kb}(\mathbf{z})A_{mc}(\mathbf{z}) - k_6 G_{kb}(\mathbf{z})G_{ma}(\mathbf{z})], \end{aligned} \tag{37}$$

$$\begin{aligned} \dot{E}_{la}(\mathbf{z}) &= \varepsilon_{lkm}\partial^z_k B_{ma}(\mathbf{z}) + g_e j^a_l + \mu_A A_{la}\\ &+\varepsilon_{abc}\varepsilon_{lkm}\hat{f}^A[k_2 A_{kb}(\mathbf{z})B'_{mc}(\mathbf{z}) + k_5 G_{kb}(\mathbf{z})E'_{mc}(\mathbf{z})], \end{aligned} \tag{38}$$

$$\begin{aligned} \dot{B}_{la}(\mathbf{z}) &= -\varepsilon_{lkm}\partial^z_k E_{ma}(\mathbf{z}) + \mathrm{i}g_m J^a_l - \mu_G G_{la}\\ &-\varepsilon_{abc}\varepsilon_{lkm}\hat{f}^A[k_2 A_{kb}(\mathbf{z})E_{mc}(\mathbf{z}) - k_5 G_{kb}(\mathbf{z})B'_{mc}(\mathbf{z})]. \end{aligned} \tag{39}$$

For $a = 0$, all terms with ε_{abc} vanish, i. e., one gets the $U(1)$ field equations.

To complete the theory of vector fields, their constraints have to be formulated (electric and magnetic Gauss law). In the canonical version of the theory these constraints need not to be postulated, but can be derived from (28)–(31) in combination with the spinor equation (25), compare for instance [13], section 8.2.

This will not be done here, because it is not along the lines of our investigation.

3. Effective Lagrangian Density

So far we have clarified the meaning of the effective canonical equations of motion of our model and brought them into a conventional form, although their mathematical and physical content exceed the content of customary electroweak gauge theories. To draw physical conclusions from these results it is advantageous to express them in the form of an effective Lagrangian, as in phenomenology the Lagrangians are the central quantities for the evaluation of the theory.

To facilitate the distinction between coordinate indices and superspin-isospin indices, we return to the η-tensor by introducing the definition

$$\eta_{abc} = \varepsilon_{abc} = \varepsilon^{abc},$$

i. e., the relation $\overline{\eta} = \mathrm{i}\eta$ holds. As in the following only η will appear, no confusion between $\overline{\eta}$ and η is possible.

To apply the Lagrange formalism, the definition of the electroweak field tensor in terms of the vector fields is required. In the literature this definition is not uniform. We follow the definition used in the treatment of gauge theories by differential forms ([15], (4.6); [16], p. 70), which reads for antisymmetric $F^a_{\mu\nu}$

$$E^a_k = -F^a_{0k}, \quad B^a_k = \frac{1}{2}\varepsilon_{kij}F^a_{ij}, \tag{40}$$

where the metric is defined by $\eta_{\mu\nu} =$ diag $(1,-1,-1,-1)$.

This definition of the fields is consistent with that used in Section 2. Furthermore, for the currents the following definitions hold:

$$j^a_\mu := \bar{\psi}\sigma^a\gamma_\mu\psi = (j^a_\mu)^+, \quad J^a_\mu := \bar{\psi}\sigma^a\gamma^5\gamma_\mu\psi = (J^a_\mu)^+,$$
$$a = 0,1,2,3, \tag{41}$$

where the minus sign in (32) is absorbed in the coupling constant.

To describe the effective field dynamics we postulate the following Lagrangian density for real vector fields with (as a preliminary condition) imaginary g_π:

$$\begin{aligned}\mathcal{L} := &-\frac{1}{4}F^a_{\mu\nu}\eta^{\mu\rho}\eta^{\nu\kappa}F^a_{\rho\kappa} \\ &+\frac{\mathrm{i}}{2}[\bar{\psi}\gamma^\mu\partial_\mu\psi + (\partial_\mu\bar{\psi})\gamma^\mu\psi] \\ &-m\bar{\psi}\psi - g_\chi A^a_\mu j^\mu_a - ig_\pi G^a_\mu j^\mu_a \\ &+\frac{1}{2}\mu_A^2 A^a_\mu\eta^{\mu\rho}A^a_\rho + \frac{1}{2}\mu_G^2 G^a_\mu\eta^{\mu\rho}G^a_\rho,\end{aligned} \tag{42}$$

where $g_x = g_\chi$, g_π takes the value g_x for $a = 0$, and g'_x for $a = 1, 2, 3$. The condition of imaginary g_π will be lifted in Section 5.

In (42) the field strength tensor is given by

$$\begin{aligned}F^a_{\mu\nu} := &\,\partial_\mu A^a_\nu - \partial_\nu A^a_\mu - \varepsilon_{\mu\nu\rho\sigma}\eta^{\rho\rho'}\eta^{\sigma\sigma'}\partial_{\rho'}G^a_{\sigma'} \\ &+\eta^{abc}(g_1 A^b_\mu A^c_\nu + g_2 G^b_\mu G^c_\nu \\ &+g_3\varepsilon_{\mu\nu\rho\sigma}\eta^{\rho\rho'}\eta^{\sigma\sigma'}A^b_{\rho'}G^c_{\sigma'}).\end{aligned} \tag{43}$$

In order to guarantee a consistent comparison with the results of the calculations in Section 2, the Lagrangian density and its associated equations of motion are exclusively expressed in terms of covariant fields. By means of the Lagrangian formalism these equations of motion can be derived from (42) and (43).

We start with the fermion equation. Its derivation is trivial. From (42) one obtains the Dirac equation

$$\begin{aligned}&\mathrm{i}\partial_\mu\gamma^\mu\psi - g_\chi A^a_\mu\sigma^a\gamma^\mu\psi - \mathrm{i}g_\pi G^a_\mu\sigma^a\gamma^5\gamma^\mu\psi \\ &+m\psi = 0.\end{aligned} \tag{44}$$

As far as the vector fields are concerned, we assume that (42) and (43) are evaluated in temporal gauge in accordance with Section 2. This gauge must be compatible with the field dynamics, even if in (42) the gauge invariance is lost, because the conjugate momenta of A^a_0 and G^a_0, $a = 0, 1, 2, 3$, vanish identically, i. e., A^a_0 and G^a_0 are no genuine independent field variables.

We first study the consequences of the field tensor definition (43).

(i) From (43) one obtains the equation for the E-fields which in temporal gauge reads

$$\begin{aligned}-E^a_k := &\,F^a_{0k} \\ = &\,\partial_0 A^a_k - \varepsilon_{0k\rho\sigma}\eta^{\rho\rho'}\eta^{\sigma\sigma'}\partial_{\rho'}G^a_{\sigma'} \\ &+g_3\eta^{abc}\varepsilon_{0k\rho\sigma}\eta^{\rho\rho'}\eta^{\sigma\sigma'}A^b_{\rho'}G^c_{\sigma'},\end{aligned} \tag{45}$$

or equivalently

$$\partial_0 A_k^a = -E_k^a + \varepsilon_{kij}\partial_i G_j^a - \frac{1}{2} g_3 \eta^{abc} \varepsilon_{kij}(A_i^b G_j^c + G_i^b A_j^c). \tag{46}$$

(ii) Similarly one obtains the equation for the B-fields from (43) in the form

$$\begin{aligned} B_k^a &:= \frac{1}{2}\varepsilon_{ijk}F_{ij}^a \\ &= \frac{1}{2}\varepsilon_{ijk}(\partial_i A_j^a - \partial_j A_i^a) \\ &\quad - \frac{1}{2}\varepsilon_{ijk}\varepsilon_{ij\rho\sigma}\eta^{\rho\rho'}\eta^{\sigma\sigma'}\partial_{\rho'}G_{\sigma'}^a \\ &\quad + \frac{1}{2}\eta^{abc}\varepsilon_{ijk}(g_1 A_i^b A_j^c - g_2 G_i^b G_j^c) \\ &\quad + \frac{1}{2} g_3 \eta^{abc}\varepsilon_{ijk}\varepsilon_{ij\rho\sigma}A_\rho^b G_\sigma^c \end{aligned} \tag{47}$$

for $i,\ j,\ k = 1,\ 2,\ 3$. Owing to this restriction the last term in (47) vanishes in temporal gauge, and after some rearrangements (47) goes over into

$$\partial_0 G_k^a = B_k^a - \varepsilon_{ijk}\partial_i A_j^a - \frac{1}{2}\eta^{abc}\varepsilon_{ijk}(g_1 A_i^b A_j^c - g_2 G_i^b G_j^c). \tag{48}$$

(iii) Next we consider the Lagrangian equations of motion for the A-fields:

$$\begin{aligned} &\partial_\mu\left(\frac{\delta\mathcal{L}}{\delta(\partial_\mu A_{\nu'}^{a'})}\right) - \frac{\delta\mathcal{L}}{\delta A_{\nu'}^{a'}} \equiv \\ &-\partial_{\mu'}\eta^{\mu\mu'}\eta^{\nu\nu'}F_{\mu\nu}^{a'} + g_\chi \eta^{\lambda\nu'} j_\lambda^{a'} - \mu_A^2 \eta^{\lambda\nu'} A_\lambda^{a'} \\ &+\frac{1}{2}\eta^{abc}(g_1 A_\mu^b \delta_{ca'}\delta_{\nu\nu'} + g_1 A_\nu^c \delta_{ba'}\delta_{\mu\nu'} \\ &+ g_3 \varepsilon_{\mu\nu\varepsilon\sigma}\delta_{\rho'\nu'}\delta_{ba'}\eta^{\varepsilon\rho'}\eta^{\sigma\sigma'}G_{\sigma'}^c)\eta^{\mu\rho}\eta^{\nu\kappa}F_{\rho\kappa}^a = 0. \end{aligned} \tag{49}$$

For $\nu' = k$ a rearrangement of (49) leads to

$$-\partial_0 E_k^{a'} = -\varepsilon_{ljk}\partial_l B_j^{a'} - \eta^{baa'}\varepsilon_{ljk}(g_1 A_l^b B_j^a + g_3 G_l^b E_j^a) + g_\chi j_k^{a'} - \mu_A^2 A_k^{a'}. \tag{50}$$

(iv) The same procedure can be performed for the G-fields. The Lagrangian equations of motion read in this case

$$\begin{aligned} &\partial_\lambda\left(\frac{\delta\mathcal{L}}{\delta(\partial_\lambda G_{\kappa'}^{a'})}\right) - \frac{\delta\mathcal{L}}{\delta G_{\kappa'}^{a'}} \equiv \\ &\frac{1}{2}\varepsilon_{\mu\nu\delta\varepsilon}\eta^{\mu\rho}\eta^{\nu\kappa}\eta^{\delta\lambda}\eta^{\varepsilon\kappa'}\partial_\lambda F_{\rho\kappa}^{a'} + \mathrm{i}g_\pi \eta^{\kappa'\lambda} J_\lambda^{a'} \\ &-\mu_G^2 \eta^{\kappa'\lambda} G_\lambda^{a'} \\ &+\frac{1}{2}\eta^{abc}(-g_2 G_\mu^b \delta_{ca'}\delta_{\nu\kappa'} - g_2 G_\nu^c \delta_{ba'}\delta_{\mu\kappa'} \\ &+ g_3 \varepsilon_{\mu\nu\varepsilon\sigma}\eta^{\varepsilon\rho'}\eta^{\sigma\sigma'}A_{\rho'}^b \delta_{\sigma'\kappa'}\delta_{ca'})\eta^{\mu\rho}\eta^{\nu\kappa}F_{\rho\kappa}^a = 0. \end{aligned} \tag{51}$$

For $\sigma' = h$, only one of the remaining indices μ,ν,ρ can adopt the value zero. Therefore the sum over μ,ν,ρ can be resolved into partial sums over ν,ρ for $\mu = 0$, μ,ρ for $\nu = 0$ and μ,ν for $\rho = 0$. This leads to

$$\begin{aligned} &-\frac{1}{2}\varepsilon_{0ijh}\partial_j F_{0i}^{a'} - \frac{1}{2}\varepsilon_{i0jh}\partial_j F_{i0}^{a'} - \frac{1}{2}\varepsilon_{ij0h}\partial_0 F_{ij}^{a'} \\ &-\mathrm{i}g_\pi J_h^{a'} + \mu_G^2 G_h^{a'} \\ &= -\frac{1}{2}(-g_2 G_l^b F_{lh}^a \eta^{aba'} - g_2 G_l^c F_{hl}^a \eta^{aa'c} \\ &\quad + g_3 \varepsilon_{\mu\nu lh} A_l^b \eta^{\mu\delta}\eta^{\nu\varepsilon'}F_{\delta\varepsilon'}^a \eta^{aba'}. \end{aligned} \tag{52}$$

If in (52) the field strength tensors are expressed by the vector fields, (52) can be reformulated in the form

$$\partial_0 B_h^{a'} = -\varepsilon_{ijh}\partial_i E_j^{a'} - \varepsilon_{jih}\eta^{baa'}(g_2 G_j^b B_i^a - g_3 A_j^b E_i^a) - \mathrm{i}g_\pi J_h^{a'} + \mu_G^2 G_h^{a'}. \tag{53}$$

In addition, the Lagrange formalism implies the derivation of the electric and magnetic Gauss law. For $\nu' = 0$ it follows the electric Gauss law from (49), while for $\sigma' = 0$ from (51) the magnetic Gauss law can be derived. It is also possible to derive pseudo-conservation laws from (49) and (51) for the currents. Because we concentrate on the discussion of the canonical equations of motion, we refer to the comment about constraints at the end of Section 2 and do not express these constraints explicitly.

In borderline cases this theory should pass into the description of conventional physics in order to be physically acceptable. To achieve this it is necessary to impose two additional conditions: The Lagrangian density should lead to equations of motion which are identical

α) with the equations of motion of a $SU(2)\otimes U(1)$ gauge theory if the magnetic vector potential G vanishes and the vector bosons are assumed to be massless;

β) with the equations of free massive electroweak bosons if their interactions are switched off.

We consider case α. In this case, (46), (48), (50) and (53) yield

$$\partial_0 A_k^a = -E_k^a, \tag{54}$$

$$B_k^a = \varepsilon_{ijk}\partial_i A_j^a + \frac{1}{2}\varepsilon_{ijk}\eta^{abc} g_1 A_i^b A_j^c,$$

$$\partial_0 E_k^a = \varepsilon_{ijk}\partial_i B_j^a + \varepsilon_{ijk}\eta^{abc} g_1 A_i^b B_j^c - g_\chi j_k^a, \tag{55}$$

$$\partial_0 B_k^a = -\varepsilon_{ijk}\partial_i E_j^a + \varepsilon_{ijk}\eta^{abc} g_3 A_i^b E_j^c. \tag{56}$$

For $a = 1, 2, 3$, these equations are identical with the equations of a $SU(2)$ gauge theory (cf. [17], eqs. (12.58), (12.59)) in temporal gauge, if the relation $g_3 = -g_1$ holds. The agreement with a $U(1)$ gauge theory for $a = 0$ is trivial.

With respect to case β we consider (49) in Lorentz-gauge switching off all interactions. This gives

$$-\partial_\mu F_a^{\mu\nu} - \mu_A^2 A_a^\nu \equiv [\partial_\mu\partial^\mu + \mu_A^2] A_a^\nu = 0, \tag{57}$$

This is the equation of a massive vector field ([3], (2.42)).

For the magnetic vector potential one obtains in this case from (51)

$$\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\partial^\rho F_a^{\mu\nu} - \mu_G^2 G_\sigma^a \equiv [\partial_\mu\partial^\mu - \mu_G^2] G_\sigma^a = 0, \tag{58}$$

where the right hand side of (58) results after some rearrangements. Obviously one has to assume $\mu_G = \mathrm{i}\bar{\mu}_G$ in order to get a physical magnetic vector boson. Only if experiments require boson velocities $\geq c$ one should deviate from this convention. Furthermore, for simplicity we assume $g_2 = g_1$.

If these conditions are incorporated in (46), (48), (50) and (53) one eventually gets the following set of equations:

$$\partial_0 A_k^a = -E_k^a + \varepsilon_{kij}\partial_i G_j^a + \frac{1}{2} g_1 \eta^{abc}\varepsilon_{kij}(A_i^b G_j^c + G_i^b A_j^c), \tag{59}$$

$$\partial_0 G_k^a = B_k^a - \varepsilon_{kij}\partial_i A_j^a - \frac{1}{2} g_1 \eta^{abc}\varepsilon_{kij}(A_i^b A_j^c - G_i^b G_j^c), \tag{60}$$

$$\partial_0 E_k^a = \varepsilon_{kij}\partial_i B_j^a + g_1\eta^{abc}\varepsilon_{kij}(A_i^b B_j^c - G_i^b E_j^c) - g_\chi j_k^a + \mu_A^2 A_k^a, \tag{61}$$

$$\partial_0 B_k^a = -\varepsilon_{kij}\partial_i E_j^a - g_1\varepsilon_{kij}\eta^{abc}(G_i^b B_j^c + A_i^b E_j^c) - \mathrm{i} g_\pi J_k^a - \bar{\mu}_G^2 G_k^a. \tag{62}$$

To compare (36) – (39) with (59) – (62), the constants in the former equations have to be fixed. In (54) of [6], their values are expressed by the formation of various scalar products of the space parts of the boson wave functions. As these scalar products (with inclusion of their regularization) are defined in an auxiliary space, they can adopt positive and negative values in contrast to the norm expressions in a physical state space.

While the algebraic structure of the boson wave functions (and, of course, of the fermion wave functions, too) is strictly set up, the space parts of these wave functions can be chosen only with a certain degree of arbitrariness which reflects the lack of information about the influence of the field theoretic vacuum on the space structure of these states. Therefore, without using selfconsistent calculation schemes for the boson wave functions, the corresponding scalar products ([6], (54)) represent parameters of the theory which can be adapted in order to get plausible results. In the present case we define

$$\begin{aligned} g_1 &= \hat{f}^A k_1 = \hat{f}^A k_2 = \hat{f}^A k_3 \\ &= \hat{f}^G k_4 = -\hat{f}^G k_5 = \hat{f}^G k_6. \end{aligned} \tag{63}$$

Theorem 1: If the relations (63) are satisfied, and the masses and coupling constants are adapted, then the set of equations (36) – (39) is identical with the set (59) – (62). The same holds for the corresponding fermion equations (35) and (44).

Addendum: The effective field theory defined by the Lagrangian density (42) is limited to a finite range of energies. Above a certain energy threshold it looses its meaning and has to be modified by formfactors etc. In this way one does not encounter the divergence difficulties of conventional field theories with Lagrangian of the type (32).

4. Transition to Effective Physical Fields

In the phenomenological treatment of the electroweak Standard Model physical fields are introduced as a consequence of isospin symmetry breaking of a corresponding $SU(2)\otimes U(1)$ gauge theory. To generate this symmetry breaking, hypothetical Higgs fields are assumed which are coupled to the boson and fermion fields and which spontaneously break local as well as global isospin invariance.

It is obvious that also in the spinor field model of Section 2 isospin symmetry breaking must be introduced in order to obtain the physical fields which are required for an appropriate description of corresponding experiments.

However, in contrast to the phenomenological procedure, in the spinor field model no Higgs fields are needed to avoid divergencies, and moreover they are an alien element in the algebraic formalism applied to this model. In the algebraic treatment, symmetry breakings are exclusively generated by changes of the representations which are related to suitable choices of the vacuum, leading to various inequivalent representations.

In the algebraic Schrödinger representation of the spinor field a change of the representation is achieved by a suitable choice of the fermion (parton) propagator. In this way isospin symmetry breaking was treated in [13, 18], for a vanishing magnetic vector potential. It is thus the task to generalize this to the case under consideration, i. e. to the Lagrangian density (42).

In the model of Section 2, CP-symmetry breaking has already been introduced by a corresponding propagator which is explicitly given in (27) in [5]. Therefore the effect of this CP-symmetry breaking on the isospin invariance has to be investigated first. According to Theorem 1, for this investigation the Lagrangian density (42) and the field strength tensor definition (43) can be used. From the mass terms in (42) it follows immediately that the local isospin symmetry is broken. Hence only the effect on the global isospin invariance has to be investigated.

Theorem 2: The Lagrangian density (42) is invariant under global isospin transformations.

Proof: We introduce the unitary transformation

$$U = \exp\left(-\mathrm{i}\frac{1}{2}\sigma_a\varepsilon_a\right) \tag{64}$$

and define

$$\mathbf{A}_\mu := \frac{1}{2}\sigma_a A^a_\mu,\ \mathbf{G}_\mu := \frac{1}{2}\sigma_a G^a_\mu,\ \mathbf{F}_{\mu\nu} := \frac{1}{2}\sigma_a F^a_{\mu\nu}. \tag{65}$$

Then for global isospin transformations the following transformation rules hold (cf. [3], section 13.2):

$$\begin{aligned} &\mathbf{A}'_\mu = U\mathbf{A}_\mu U^+, \quad \mathbf{G}'_\mu = U\mathbf{G}_\mu U^+, \\ &\psi' = U\psi, \quad \bar{\psi}' = \bar{\psi}U^+. \end{aligned} \tag{66}$$

To represent the definition of the field strength tensor in this form we multipy (43) with $(1/2)\sigma_a$ and sum over a. With the group theoretical commutation relations

$$\begin{aligned} \mathrm{i}\frac{1}{2}\eta^{abc}\sigma_a X^b_\mu Y^c_\nu &= \left[\frac{1}{2}\sigma_b X^b_\mu, \frac{1}{2}\sigma_c Y^c_\nu\right]_- \\ &= [\mathbf{X}_\mu, \mathbf{Y}_\nu]_- \end{aligned} \tag{67}$$

one obtains for (43) the expression

$$\begin{aligned} &\mathbf{F}_{\mu\nu} := (\partial_\mu \mathbf{A}_\nu - \partial_\nu \mathbf{A}_\mu) - \varepsilon_{\mu\nu\rho\sigma}\partial^\rho \mathbf{G}^\sigma \\ &-\mathrm{i}g_1([\mathbf{A}_\mu \mathbf{A}_\nu]_- - [\mathbf{G}_\mu \mathbf{G}_\nu]_- - \varepsilon_{\mu\nu\rho\sigma}[\mathbf{A}^\rho \mathbf{G}^\sigma]_-), \end{aligned} \tag{68}$$

and from this expression it follows directly that

$$\mathbf{F}'_{\mu\nu} = U\mathbf{F}_{\mu\nu}U^+. \tag{69}$$

Having derived this relation, we can represent the Lagrangian density (42) in the form

$$\begin{aligned} \mathcal{L} = &-\frac{1}{2}T_r[\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}] + 2g'_e\bar{\psi}\mathbf{A}_\mu\gamma^\mu\psi \\ &+ 2\mathrm{i}g'_m\bar{\psi}\mathbf{G}_\mu\gamma^5\gamma^\mu\psi + \frac{1}{2}\mu_A T_r[\mathbf{A}_\mu\mathbf{A}^\mu] \\ &- \frac{1}{2}\bar{\mu}_G T_r[\mathbf{G}_\mu\mathbf{G}^\mu] + \mathcal{L}_{U(1)} + \mathrm{i}[\bar{\psi}\gamma^\mu\partial_\mu\psi \\ &+ (\partial_\mu\bar{\psi})\gamma^\mu\psi] + m\bar{\psi}\psi. \end{aligned} \tag{70}$$

Obviously this density is invariant under global isospin transformations. $\diamondsuit$

In addition to this result it can be shown that the CP-symmetry breaking propagator in (27) in [5] is invariant under global isospin transformations too. The algebraic part of this propagator contains the superspin-isospin matrices γ^5 and $\gamma^5\gamma^0$, which are represented in the form $\gamma^5 = \sigma^1_{A_1A_2}\sigma^0_{a_1a_2}$ and $\gamma^5\gamma^0 = -\mathrm{i}\sigma^2_{A_1A_2}\sigma^0_{a_1a_2}$, where $\sigma^0_{a_1a_2}$ stands for the isospin part. This matrix is the unit matrix in isospin space, and hence this matrix and so the propagator are invariant under global isospin transformations. Furthermore, as the spinor field propagator acts as a relativistic potential in the mass eigenvalue equations of bosons and fermions, its global isospin invariance leads to degenerate mass matrices in isospin space. This fact is expressed by the mass matrices in the Lagrangian density (42).

On the other hand, in phenomenology the Higgs fields break the global isospin invariance which manifests itself by nondegenerate (phenomenological) boson and fermion mass matrices. Therefore the propagator in (27) in [5] is not appropriate to reproduce the effect of the Higgs fields within the algebraic formalism. An appropriate propagator must explicitly break global isospin invariance in order to remove the degeneracy of the mass matrices and thus to come to an agreement with phenomenology.

In the following we concentrate on the calculation of the nondegenerate mass matrix for bosons, but in this context we do not repeat those calculations which lead to the degenerate boson mass matrix of (109) and (110) in [6]. A comment about the fermion mass matrix will be given below.

In functional space the *full* term which implicitly contains the effective boson mass matrix is given by

$$m^b_{kl} b_k \partial^b_l + M^b_{kl} b_k \partial_l \equiv 2R^k_{II_1} m^f_{I_1 I_2} C^l_{I_2 I} b_k \partial^b_l - 6W_{I_1 I_2 I_3 I_4} F_{I_4 K} R^k_{KI_1} C^l_{I_2 I_3} b_k \partial^b_l, \qquad (71)$$

where the first term on both sides stems from the spinorial mass matrix of (1) in [6], while the second term contains the fermion propagator of the spinor field. We now assume that this propagator breaks the CP-symmetry as well as isospin symmetry. Provided that the symmetry breaking admixtures are small, their contributions to the propagator are additive, as higher-order terms of their Taylor expansion can be neglected. Therefore the results of their separate calculation can be linearly superposed.

We thus decompose the propagator of formula (71) into two parts:

$$F := F^0 + F^1, \qquad (72)$$

where F^0 is defined by the CP-symmetry violating propagator in (27) in [5], while F^1 represents the isospin symmetry breaking part. The calculations of [6] are referred to F^0, and one obtains from (40)–(48) of [6] with inclusion of the transformations (2)

$$\begin{aligned} m^b_{kl} b_k \partial^b_l + M^b_{kl} b_k \partial^b_l &\equiv 2R^k_{II_1} m^f_{I_1 I_2} C^l_{I_2 I} b_k \partial^b_l - 6W_{I_1 I_2 I_3 I_4} F_{I_4 K} R^k_{KI_1} C^l_{I_2 I_3} b_k \partial^b_l \\ &\equiv \mathrm{i} \int \mathrm{d}^3 z\, b^E_{la}(\mathbf{z})[c_2 - \hat f^A c_4]\partial^A_{la}(\mathbf{z}) \\ &\quad - \mathrm{i} \int \mathrm{d}^3 z\, b^B_{la}(\mathbf{z})[c_3 - \hat f^G c_4]\partial^G_{la}(\mathbf{z}) \\ &=: \mathrm{i} \int \mathrm{d}^3 z \mu_A^2 b^E_{la}(\mathbf{z})\partial^A_{la}(\mathbf{z}) - \mathrm{i} \int \mathrm{d}^3 z \mu_G^2 b^B_{la}(\mathbf{z})\partial^G_{la}(\mathbf{z}). \end{aligned} \qquad (73)$$

The mass matrices in (73) are degenerate in isospin space in agreement with the mass terms in (11) and (12).

For the calculation of (71) the single time propagator is required. The corresponding isospin symmetry breaking part F^1 can be written in the general form

$$F^1 := \lambda_{i_1} (\gamma^0 \gamma^3)_{\kappa_1 \kappa_2} [h_{i_1}(\mathbf{u}) C_{\beta_1 \beta_2} + h^k_{i_1}(\mathbf{u})(\gamma^k C)_{\beta_1 \beta_2}], \qquad (74)$$

where $\mathbf{u}$ is the relative coordinate $\mathbf{r}_1 - \mathbf{r}_2$. In (8.51) in [13] the construction of F^1 is explained starting from the general explicite form (8.49) in [13]. As we are only interested in the algebraic evaluation of (71), we suppress the explicit representation of the coordinate parts h, h^k for the sake of brevity.

To evaluate the isospin symmetry breaking term, we define

$$\mathcal{H}^2_b(F^n) := -6W_{I_1 I_2 I_3 I_4} F^n_{I_4 K} R^k_{KI_1} C^l_{I_2 I_3} b_k \partial^b_l, \qquad (75)$$

which is a generalization of (46) in [6] for various propagators F^n, $n = 0, 1, \ldots$, or for different pieces of one propagator in accordance with (72).

The formula (47) in [6] is thus identical with $\mathcal{H}^2_b(F^0)$. Hence, if we replace F^0 in (47) in [6] by F^1, we can use this formula as the starting point of our calculation. This formula reads

$$\begin{aligned} \mathcal{H}^2_b(F^1) &= g \int \mathrm{d}^3 r_1\, \mathrm{d}^3 r\, \mathrm{d}^3 k\, \mathrm{d}^3 k' \\ &\cdot \Big\{ \lambda_{i_1} \sum_h \Big[(\gamma^0 \upsilon^h)_{\beta_1\beta_2} (\upsilon^h C)_{\beta_3\beta_4} \delta_{\rho_1\rho_2} \gamma^5_{\rho_3\rho_4} - (\gamma^0 \upsilon^h)_{\beta_1\beta_3} (\upsilon^h C)_{\beta_2\beta_4} \delta_{\rho_1\rho_3} \gamma^5_{\rho_2\rho_4} - (\gamma^0 \upsilon^h)_{\beta_1\beta_4} (\upsilon^h C)_{\beta_3\beta_2} \delta_{\rho_1\rho_4} \gamma^5_{\rho_3\rho_2} \Big] \Big\} \\ &\cdot \sum_{i_4} \lambda_{i_4} (\gamma^0\gamma^3)_{\rho_4\kappa} \delta_{i_4 i'} \Big[h_{i_4}(\mathbf{r}_1 - \mathbf{r}) C_{\beta_4\alpha} + h^k_{i_4}(\mathbf{r}_1 - \mathbf{r})(\gamma^k C)_{\beta_4\alpha} \Big] \\ &\cdot (T^{a'} + S^{a'})_{\rho_2\rho_3} \Big[\hat f^A(0|m)_{\beta_2\beta_3} \partial^A_{ma'}(\mathbf{k}') + \hat f^E(0|m)_{\beta_2\beta_3} \partial^E_{ma'}(\mathbf{k}') + \hat f^B(0|m)_{\beta_2\beta_3} \partial^B_{ma'}(\mathbf{k}') + \hat f^G(0|m)_{\beta_2\beta_3} \partial^G_{ma'}(\mathbf{k}') \Big] \\ &\cdot \exp[-\mathrm{i}\mathbf{k}'\mathbf{r}_1] (T^a + S^a)^+_{\kappa\rho_1} \Big[r^A_{i_1 i'}(\mathbf{r}_1 - \mathbf{r}|l)_{\alpha\beta_1} b^A_{la}(\mathbf{k}) + r^E_{i_1 i'}(\mathbf{r}_1 - \mathbf{r}|l)_{\alpha\beta_1} b^E_{la}(\mathbf{k}) + r^B_{i_1 i'}(\mathbf{r}_1 - \mathbf{r}|l)_{\alpha\beta_1} b^B_{la}(\mathbf{k}) \\ &\qquad + r^G_{i_1 i'}(\mathbf{r}_1 - \mathbf{r}|l)_{\alpha\beta_1} b^G_{la}(\mathbf{k}) \Big] \exp[\mathrm{i}\mathbf{k}\frac{1}{2}(\mathbf{r}_1 + \mathbf{r})]. \end{aligned} \qquad (76)$$

Next, by $(\mathbf{r}_1-\mathbf{r})=\mathbf{u}$ and $(\mathbf{r}_1+\mathbf{r})1/2=\mathbf{z}$ we introduce center of mass coordinates. For these coordinates we obtain $\exp(-\mathrm{i}\mathbf{k}'\mathbf{r}_1)=\exp[-\mathrm{i}\mathbf{k}'(\mathbf{z}+\mathbf{u}/2)]$, which we approximately replace by $\exp(-\mathrm{i}\mathbf{k}'\mathbf{z})$ as the propagator functions and the dual boson functions are strongly concentrated around the origin $\mathbf{u}=0$, and one can consider them as test functions and assume $\exp(-\mathrm{i}\mathbf{k}'\mathbf{u})\approx 1$. Then the integration over $\mathbf{k}$ and $\mathbf{k}'$ can be performed, leading to the equivalent expression

$$\begin{aligned}\mathcal{H}_b^2(F^1) &= g\int \mathrm{d}^3z\,\mathrm{d}^3u\\
&\cdot\Big\{\lambda_{i_1}\sum_h\Big[(\gamma^0\upsilon^h)_{\beta_1\beta_2}(\upsilon^hC)_{\beta_3\beta_4}\delta_{\rho_1\rho_2}\gamma^5_{\rho_3\rho_4}-(\gamma^0\upsilon^h)_{\beta_1\beta_3}(\upsilon^hC)_{\beta_2\beta_4}\delta_{\rho_1\rho_3}\gamma^5_{\rho_2\rho_4}-(\gamma^0\upsilon^h)_{\beta_1\beta_4}(\upsilon^hC)_{\beta_3\beta_2}\delta_{\rho_1\rho_4}\gamma^5_{\rho_3\rho_2}\Big]\Big\}\\
&\cdot\sum_{i_4}\lambda_{i_4}(\gamma^0\gamma^3)_{\rho_4\kappa}\delta_{i_4i'}\Big[h_{i_4}(\mathbf{u})C_{\beta_4\alpha}+h^k_{i_4}(\mathbf{u})(\gamma^kC)_{\beta_4\alpha}\Big](T^{a'}+S^{a'})_{\rho_2\rho_3}\\
&\cdot\Big[\hat{f}^A(0|m)_{\beta_2\beta_3}\partial^A_{ma'}(\mathbf{z})+\hat{f}^E(0|m)_{\beta_2\beta_3}\partial^E_{ma'}(\mathbf{z})+\hat{f}^B(0|m)_{\beta_2\beta_3}\partial^B_{ma'}(\mathbf{z})+\hat{f}^G(0|m)_{\beta_2\beta_3}\partial^G_{ma'}(\mathbf{z})\Big](T^a+S^a)^+_{\kappa\rho_1}\\
&\cdot\Big[r^A_{i_1i'}(\mathbf{u}|l)_{\alpha\beta_1}b^A_{la}(\mathbf{z})+r^E_{i_1i'}(\mathbf{u}|l)_{\alpha\beta_1}b^E_{la}(\mathbf{z})+r^B_{i_1i'}(\mathbf{u}|l)_{\alpha\beta_1}b^B_{la}(\mathbf{z})+r^G_{i_1i'}(\mathbf{u}|l)_{\alpha\beta_1}b^G_{la}(\mathbf{z})\Big]. \qquad (77)\end{aligned}$$

The algebraic evaluation of this expression can be done exactly, but is rather extensive. For brevity we give only the result of this calculation, which reads with transformations (2)

$$\begin{aligned}\mathcal{H}_b^2(F^1) =& 192\mathrm{i}\int \mathrm{d}^3u\sum_{i_1i'}\lambda_{i_1}\lambda_{i'}\Big[r^E_{i_1i'}(\mathbf{u})h_{i'}(\mathbf{u})\Big]\hat{f}^A\int\mathrm{d}^3z\Big[b^E_{l0}(\mathbf{z})\partial^A_{l3}(\mathbf{z})+b^E_{l3}(\mathbf{z})\partial^A_{l0}(\mathbf{z})+\mathrm{i}b^E_{l1}(\mathbf{z})\partial^A_{l2}(\mathbf{z})-\mathrm{i}b^E_{l2}(\mathbf{z})\partial^A_{l1}(\mathbf{z})\Big]\\
&+192\mathrm{i}\int \mathrm{d}^3u\sum_{i_1i'}\lambda_{i_1}\lambda_{i'}\Big[r^E_{i_1i'}(\mathbf{u})h_{i'}(\mathbf{u})\Big]\hat{f}^G\int\mathrm{d}^3z\Big[b^B_{l0}(\mathbf{z})\partial^G_{l3}(\mathbf{z})+b^B_{l3}(\mathbf{z})\partial^G_{l0}(\mathbf{z})+\mathrm{i}b^B_{l1}(\mathbf{z})\partial^G_{l2}(\mathbf{z})-\mathrm{i}b^B_{l2}(\mathbf{z})\partial^G_{l1}(\mathbf{z})\Big], \quad (78)\end{aligned}$$

and which leads to a modified boson mass matrix.

This result is the most important effect of isospin symmetry breaking in the sytem, as it is the starting point for the introduction of effective physical boson fields. In addition, the modified propagator influences the fermion mass matrix and the current coupling to the boson fields, as can be read off from the definition of the various contributions to the functional energy expression (22)–(24) in [6]. But for our intended application these modifications are of minor interest, as will be seen in the next section. Hence we suppress them.

Under these presuppositions the functional energy operator (1) is modified to give

$$\begin{aligned}\tilde{\mathcal{H}} &= \mathcal{H}_f+\mathcal{H}_b^1+\mathcal{H}_b^2(F^0)+\mathcal{H}_b^2(F^1)\\ &\quad+\mathcal{H}_b^3+\mathcal{H}_{bf}^1+\mathcal{H}_{bf}^2.\end{aligned} \qquad (79)$$

By means of this energy operator the effective classical equations of motion can be derived. In comparison with the isospin-invariant energy operator (1) and its equations of motion (9)–(13) or (59)–(62), respectively, only the field equations for the electric and magnetic fields are modified, while the equations for the electric and magnetic vector potentials remain unchanged. Hence we derive only the former equations explicitly. But owing to the varying contributions of (78) to the sets of equations for $a=1$, 2 and $a=3$, 0, it is suitable to treat these sets separately. For $a=1$, 2 one gets

$$\begin{aligned}\partial_0E^a_k &= \varepsilon_{\mathrm{k}ij}\partial_iB^a_j+g_1\eta^{abc}\varepsilon_{\mathrm{k}ij}(A^b_iB^c_j-G^b_lE^a_j)\\ &\quad-g_\chi j^a_k+\mu_A^2A^a_k-a_A\sigma^2_{ab}A^b_k,\end{aligned} \qquad (80)$$

$$\begin{aligned}\partial_0B^a_k &= -\varepsilon_{\mathrm{k}ij}\partial_iE^a_j-g_1\varepsilon_{\mathrm{k}ij}\eta^{abc}(G^b_iB^c_j+A^b_iE^c_j)\\ &\quad-\mathrm{i}g_\pi J^a_k-\bar{\mu}_G^2G^a_k-a_G\sigma^2_{ab}G^b_k,\end{aligned} \qquad (81)$$

for the electric and magnetic fields, while for $a=3$, 0 one obtains

$$\begin{aligned}\partial_0E^a_k &= \varepsilon_{\mathrm{k}ij}\partial_iB^a_j+g_1\eta^{abc}(\delta_{a0}-1)\varepsilon_{\mathrm{k}ij}(A^b_iB^c_j-G^b_lE^a_j)\\ &\quad-g_\chi j^a_k+\mu_A^2A^a_k+a'_A\sigma^1_{ab}A^b_k,\end{aligned} \qquad (82)$$

$$\begin{aligned}\partial_0B^a_k &= -\varepsilon_{\mathrm{k}ij}\partial_iE^a_j-g_1\eta^{abc}(\delta_{a0}-1)\varepsilon_{\mathrm{k}ij}(G^b_iB^c_j+A^b_iE^c_j)\\ &\quad-\mathrm{i}g_\pi J^a_k-\bar{\mu}_G^2G^a_k-a'_G\sigma^1_{ab}G^b_k,\end{aligned} \qquad (83)$$

as the completion of (61) and (62) in the case of isospin symmetry breaking of the vacuum. Note that σ_{ab} can and is to be defined on the set $a,b=3$, 0.

In these equations the nondiagonal form of the mass matrices shows that their vector fields are not identifiable with observable fields, because in scattering processes no free vector fields result asymptotically. The transition to observable fields can only be achieved by application of generalized Weinberg transformations, which for (80) and (81) simultaneously lead to definite electric charges of the vector bosons. (In the conventional theory such a linking between Weinberg transformations and charge eigenstates does not exist.)

In view of the large number of field variables it is not advisable to use the conventional symbols of these fields. Thus for the electric part we rename them in the following way:

$$\begin{aligned}&\tilde{\mathbf{E}}_1 := \mathbf{E}^{W^-}, \tilde{\mathbf{E}}_2 := \mathbf{E}^{W^+}, \tilde{\mathbf{E}}_3 := \mathbf{E}^{Z}, \tilde{\mathbf{E}}_0 := \mathbf{E}^{A},\\ &\tilde{\mathbf{A}}_1 := \mathbf{W}^-,\ \tilde{\mathbf{A}}_2 := \mathbf{W}^+,\ \tilde{\mathbf{A}}_3 := \mathbf{Z},\ \ \tilde{\mathbf{A}}_0 := \mathbf{A},\end{aligned} \tag{84}$$

while for the magnetic part we define

$$\begin{aligned}&\tilde{\mathbf{B}}_1 := \mathbf{B}^{M^-}, \tilde{\mathbf{B}}_2 := \mathbf{B}^{M^+}, \tilde{\mathbf{B}}_3 := \mathbf{B}^{X}, \tilde{\mathbf{B}}_0 := \mathbf{B}^{G},\\ &\tilde{\mathbf{G}}_1 := \mathbf{M}^-,\ \tilde{\mathbf{G}}_2 := \mathbf{M}^+,\ \tilde{\mathbf{G}}_3 := \mathbf{X},\ \ \tilde{\mathbf{G}}_0 := \mathbf{G},\end{aligned} \tag{85}$$

where the symbols for the magnetic fields are freely adapted from the electric fields.

By definition these fields are assumed to diagonalize the mass matrices and are thus the physical fields. They are related to the original fields by the transformations

$$E_k^a = y_{ab}\tilde{E}_k^b, \quad B_k^a = y_{ab}\tilde{B}_k^b, \quad a = 1,2, \tag{86}$$

and

$$E_k^a = z_{ab}\tilde{E}_k^b, \quad B_k^a = z_{ab}\tilde{B}_k^b, \quad a = 3,0, \tag{87}$$

where y_{ab} and z_{ab} are given by the unitary or orthogonal matrices, respectively:

$$y_{ab} = 2^{-1/2}\begin{pmatrix} 1 & 1\\ -\mathrm{i} & \mathrm{i}\end{pmatrix}, \quad z_{ab} = \begin{pmatrix}\cos\Theta & \sin\Theta\\ -\sin\Theta & \cos\Theta\end{pmatrix}. \tag{88}$$

The same transformations hold for the vector potentials:

$$A_k^a = y_{ab}\tilde{A}_k^b, \quad G_k^a = y_{ab}\tilde{G}_k^b, \quad a = 1,2, \tag{89}$$

and

$$A_k^a = z_{ab}\tilde{A}_k^b, \quad G_k^a = z_{ab}\tilde{G}_k^b, \quad a = 3,0. \tag{90}$$

The consistency of the simultaneous transformation of fields and vector potentials can be proven by using the Hamiltonian equations of motion for the complex vector fields (84) and (85) and transforming them back to the original fields and potentials (cf. [13], section 8.1 and [22], (2.63), (2.84a), (2.84b)). If y_{ab}^+ is applied to (80) and (81), these equations can be rewritten in the following form:

$$\begin{aligned}\partial_0\tilde{E}_k^a = {}&\varepsilon_{kij}\partial_i\tilde{B}_j^a + g_1 y_{aa'}^+\eta^{a'bc}\varepsilon_{kij}(A_i^bB_j^c - G_l^bE_j^a)\\ &- g_\chi y_{aa'}^+ j_k^{a'} + (\mu_A^2\sigma_{ab}^0 + a_A\sigma_{ab}^3)\tilde{A}_k^b,\end{aligned} \tag{91}$$

$$\begin{aligned}\partial_0\tilde{B}_k^a = {}&-\varepsilon_{kij}\partial_i\tilde{E}_j^a - g_1 y_{aa'}^+\eta^{a'bc}\varepsilon_{kij}(G_i^bB_j^c + A_i^bE_j^c)\\ &- \mathrm{i}g_\pi y_{aa'}^+ J_k^{a'} - (\bar{\mu}_G^2\sigma_{ab}^0 - a_G\sigma_{ab}^3)\tilde{G}_k^b,\end{aligned} \tag{92}$$

while, after application of z_{ab}^T, (82) and (83) yield

$$\begin{aligned}\partial_0\tilde{E}_k^a = {}&\varepsilon_{kij}\partial_i\tilde{B}_j^a + g_1 z_{aa'}^T\eta^{a'bc}(\delta_{a0} - 1)\\ &\cdot\varepsilon_{kij}(A_i^bB_j^c - G_l^bE_j^a) - g_\chi z_{aa'}^T j_k^{a'}\\ &+ (\mu_A^2\sigma_{ab}^0 - 2(\cos\Theta)^2 a_A\sigma_{ab}^3)\tilde{A}_k^b,\end{aligned} \tag{93}$$

$$\begin{aligned}\partial_0\tilde{B}_k^a = {}&-\varepsilon_{kij}\partial_i\tilde{E}_j^a - g_1 z_{aa'}^T\eta^{a'bc}(\delta_{a0} - 1)\\ &\cdot\varepsilon_{kij}(G_i^bB_j^c + A_i^bE_j^c) - \mathrm{i}g_\pi z_{aa'}^T J_k^{a'}\\ &- (\bar{\mu}_G^2\sigma_{ab}^0 + 2(\cos\Theta)^2 a_G\sigma_{ab}^3)\tilde{G}_k^b.\end{aligned} \tag{94}$$

In these equations the nonlinear terms have not been transformed to the new field variables, because the nonlinear terms mix the two sectors. To transform the latter terms as well, one has to return to the four-dimensional representation which is anticipated in the definitions (84) and (85) for the fields and which will be applied below.

To obtain the diagonal form of the mass matrices in (93) and (94) the condition $\cos^2\Theta = \sin^2\Theta$ has to be satisfied, which gives the Weinberg angle $\Theta = 45°$. The latter result is a consequence of the equal mass values of the vector boson singlet and triplet states in (11) and (12). This is in accordance with the result of the state calculations for single electric and magnetic bosons in [5]. In a selfconsistent calculation of their wave functions, this degeneracy may be removed, but in the case under consideration this leads to the idealized form of the Weinberg transformation z_{ab} with the Weinberg angle $\Theta = 45°$.

Furthermore, apart from diagonalizing the mass matrices in (91) – (94), the unitary transformation (86) are identical with that transformation in phenomenological theory which leads to the charged vector boson states. By application of this transformation to the microscopic boson wave functions defined in (27) and (28) in [6], it can be easily verified that these wave functions are eigenfunctions of the microscopic charge operator in (6.101) – (6.103) in [13] with correct eigenvalues.

To return to a four-dimensional notation of the isospin states in (91) – (94) we define the four-dimensional transformation matrix

$$t_{nm} = \begin{pmatrix} y_{ab} & 0 \\ 0 & z_{ab} \end{pmatrix} \tag{95}$$

and obtain for the complete set of field equations the following transformed version:

$$\partial_0 \tilde{A}^a_k = -\tilde{E}^a_k + \varepsilon_{kij}\partial_i \tilde{G}^a_j + \frac{1}{2} g_1 \tilde{\eta}^{alh} \varepsilon_{kij}(\tilde{A}^l_i \tilde{G}^h_j + \tilde{G}^l_i \tilde{A}^h_j), \tag{96}$$

$$\partial_0 \tilde{G}^a_k = \tilde{B}^a_k - \varepsilon_{kij}\partial_i \tilde{A}^a_j - \frac{1}{2} g_1 \tilde{\eta}^{alh} \varepsilon_{kij}(\tilde{A}^l_i \tilde{A}^h_j - \tilde{G}^l_i \tilde{G}^h_j), \tag{97}$$

$$\partial_0 \tilde{E}^a_k = \varepsilon_{kij}\partial_i \tilde{B}^a_j + g_1 \tilde{\eta}^{alh} \varepsilon_{kij}(\tilde{A}^l_i \tilde{B}^h_j - \tilde{G}^l_i \tilde{E}^h_j) - g_\chi \hat{j}^a_k + [\mu_A^2 I_{aa'} + a_A(\gamma^5\gamma^3)_{aa'}]\tilde{A}^{a'}_k, \tag{98}$$

$$\partial_0 \tilde{B}^a_k = -\varepsilon_{kij}\partial_i \tilde{E}^a_j - g_1 \varepsilon_{kij} \tilde{\eta}^{alh}(\tilde{G}^l_i \tilde{B}^h_j + \tilde{A}^l_i \tilde{E}^h_j) - \mathrm{i} g_\pi \hat{J}^a_k - [\mu_G^2 I_{aa'} - a_G(\gamma^5\gamma^3)_{aa'}]\tilde{G}^{a'}_k \tag{99}$$

with the definitions

$$\tilde{\eta}^{nlh} := t^+_{nm} \eta^{mbc} t_{bl} t_{ch},\ \hat{j}^a_k := t^+_{ab} j^a_k,\ \hat{J}^a_k := t^+_{ab} J^a_k, \tag{100}$$

where it has to be taken into account that η^{mbc} vanishes if one of the superscripts is zero. Explicit calculation yields:

$$\begin{aligned}
\tilde{\eta}^{1lh} &= 2\delta_{n1}\mathrm{i}(\delta_{l2}\delta_{h3} - \delta_{l3}\delta_{h2} + \delta_{l2}\delta_{h0} - \delta_{l0}\delta_{h2}),\\
\tilde{\eta}^{2lh} &= 2\delta_{n2}\mathrm{i}(-\delta_{l1}\delta_{h3} + \delta_{l3}\delta_{h1} - \delta_{l1}\delta_{h0} + \delta_{l0}\delta_{h1}),\\
\tilde{\eta}^{3lh} &= 2\delta_{n3}\mathrm{i}(\delta_{l1}\delta_{h2} - \delta_{l2}\delta_{h1}),\\
\tilde{\eta}^{0lh} &= 2\delta_{n0}\mathrm{i}(\delta_{l1}\delta_{h2} - \delta_{l2}\delta_{h1}).
\end{aligned} \tag{101}$$

Finally we treat the fermion equation. We start with equation (35) or equivalently with (44). Transforming to the physical fields (84) and (85) one obtains

$$\begin{aligned}
&[-\mathrm{i}\gamma^\mu \partial_\mu + m]\psi_a + \frac{1}{2}\gamma^k 2^{-1/2}\Big[\tilde{A}^1_k g(\sigma^1 - \mathrm{i}\sigma^2)_{ab}\\
&+\tilde{A}^2_k g(\sigma^1 + \mathrm{i}\sigma^2)_{ab} + \tilde{A}^3_k(\sigma^3 g + \sigma^0 g')_{ab}\\
&+\tilde{A}^0_k(\sigma^3 g - \sigma^0 g')_{ab}\Big]\psi_b + \mathrm{i}\frac{1}{2}(\gamma^k\gamma^5)2^{-1/2}\\
&\cdot\Big[\tilde{G}^1_k g(\sigma^1 - \mathrm{i}\sigma^2)_{ab} + \tilde{G}^2_k g(\sigma^1 + \mathrm{i}\sigma^2)_{ab}\\
&+\tilde{G}^3_k(\sigma^3 g + \sigma^0 g')_{ab} + \tilde{G}^0_k(\sigma^3 g - \sigma^0 g')_{ab}\Big]\psi_b = 0.
\end{aligned} \tag{102}$$

In this equation only the electric vector potentials correspond to the conventional theory. To verify this equivalence it has to be noted that by group theoretical construction the eigenvalues and eigenstates of the composite fermions can be determined, but not their numeration. The latter is quite arbitrary. By definition in S-representation of equation (35) a numeration is chosen which agrees with the phenomenological one. This means that $\psi_1 \equiv \nu$ and $\psi_2 \equiv e^-$. Then, owing to the idealized Weinberg angle, one must assume $g' = g$ in order to obtain the correct operator $\sigma^3 - \sigma^0$ which governs the coupling of the photons to electrons in the $\tilde{A}^0_k$-term. For this Weinberg angle and value of g' the remaining $\tilde{A}$-terms for $a = 1, 2, 3$ correspond to the conventional theory (cf. [2], (22.30) – (22.32)).

5. Modification of β-Decay Processes

The conventional electroweak Standard Model contains a great number of different elementary interactions (cf. the schedule in [19], Table 14.1). If this Standard Model is extended to comprise the magnetic vector bosons, then the number of elementary interactions is correspondingly increased. To exemplify these changes we concentrate on the discussion of one type of interaction which led to the discovery of the weak forces in the past and which is of present interest, namely the β-decay.

The three basic processes in nuclear β-decay are

$$\begin{aligned}
&\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \overline{\nu}_\mathrm{e},\\
&\mathrm{p} \to \mathrm{n} + \mathrm{e}^+ + \nu_\mathrm{e},\\
&\mathrm{p} + \mathrm{e}^- \to \mathrm{n} + \nu_\mathrm{e},
\end{aligned} \tag{103}$$

which in the quark picture are replaced by

$$\begin{aligned}
&\mathrm{d} \to \mathrm{u} + \mathrm{e}^- + \overline{\nu}_\mathrm{e},\\
&\mathrm{u} \to \mathrm{d} + \mathrm{e}^+ + \nu_\mathrm{e},\\
&\mathrm{u} + \mathrm{e}^- \to \mathrm{d} + \nu_\mathrm{e},
\end{aligned} \tag{104}$$

where the last process in (104) describes nuclear electron capture. Although the latter process starts with bound electron states (which are modified in the presence of magnetic photons), the transition rates are closely related between all these processes [20]. Hence it is admissible to study the most simple representative of this family of processes in the following.

In this paper the theoretical basis for the treatment of these phenomena is assumed to be given by the canonical equations of motion for the physical fields of bosons as well as of fermions, which have been derived in the preceding section. These effective canonical equations of motion for the physical fields are rooted in the microscopic spinor theory and are thus basic. But from a phenomenological point of view these equations are not adapted to the discussion of the above processes.

In the Standard Model electroweak processes are described by perturbation theory. Therefore, to investigate electroweak processes, (96) – (99) and (102) have to be rearranged in order to allow a perturbation theoretic interpretation. In addition, the quark states must be included in this formalism to guarantee a complete description of the processes (104). Without going back to the microscopic theory we introduce quark states by suitable definitions, since a microscopic derivation would exceed the scope of this paper. For a microscopic derivation we refer to [13], chapter 7.

Equations (96) – (99) and (102) are classical equations, whereas perturbation theory refers to quantum processes. We use a semiclassical method to arrive at expressions which can be interpreted as perturbation theoretic matrix elements. We define the following auxiliary quantities:

$$\begin{aligned}
\mathbf{N}_A^a &:= \frac{1}{2} g_1 \tilde{\eta}^{alh} \varepsilon_{kij} (\tilde{A}_i^l \tilde{G}_j^h + \tilde{G}_i^l \tilde{A}_j^h), \\
\mathbf{N}_G^a &:= -\frac{1}{2} g_1 \tilde{\eta}^{alh} \varepsilon_{kij} (\tilde{A}_i^l \tilde{A}_j^h - \tilde{G}_i^l \tilde{G}_j^h), \\
\mathbf{N}_E^a &:= g_1 \tilde{\eta}^{alh} \varepsilon_{kij} (\tilde{A}_i^l \tilde{B}_j^h - \tilde{G}_i^l \tilde{E}_j^h) - g_\chi \hat{j}_k^a, \\
\mathbf{N}_B^a &:= -g_1 \tilde{\eta}^{alh} \varepsilon_{kij} (\tilde{G}_i^l \tilde{B}_j^h + \tilde{A}_i^l \tilde{E}_j^h) - \mathrm{i} g_\pi \hat{J}_k^a,
\end{aligned} \tag{105}$$

and observe that the electric and magnetic mass tensors are diagonal:

$$\begin{aligned}
[\mu_A^2 I_{aa'} + a_A (\gamma^5 \gamma^3)_{aa'}] \tilde{A}_k^{a'} &= m_A^a \tilde{A}_k^a, \\
[\mu_G^2 I_{aa'} - a_G (\gamma^5 \gamma^3)_{aa'}] \tilde{G}_k^{a'} &= m_G^a \tilde{G}_k^a.
\end{aligned} \tag{106}$$

Then (96) – (99) can be rewritten in the form

$$\partial_0 \tilde{A}_k^a = -\tilde{E}_k^a + \varepsilon_{kij} \partial_i \tilde{G}_j^a + N_{A,k}^a, \tag{107}$$

$$\partial_0 \tilde{G}_k^a = \tilde{B}_k^a - \varepsilon_{kij} \partial_i \tilde{A}_j^a + N_{G,k}^a, \tag{108}$$

$$\partial_0 \tilde{E}_k^a = \varepsilon_{kij} \partial_i \tilde{B}_j^a + m_A^a \tilde{A}_k^a + N_{E,k}^a, \tag{109}$$

$$\partial_0 \tilde{B}_k^a = -\varepsilon_{kij} \partial_i \tilde{E}_j^a - m_G^a \tilde{G}_k^a + N_{b,k}^a. \tag{110}$$

These equations can exactly be transformed into the set of equations

$$\frac{\partial^2}{\partial t^2} \tilde{\mathbf{A}}^a + \nabla \times \nabla \times \tilde{\mathbf{A}}^a + m_A^a \tilde{\mathbf{A}}^a = \mathbf{R}_A^a, \tag{111}$$

$$\frac{\partial^2}{\partial t^2} \tilde{\mathbf{G}}^a + \nabla \times \nabla \times \tilde{\mathbf{G}}^a - m_G^a \tilde{\mathbf{G}}^a = \mathbf{R}_G^a, \tag{112}$$

with the definitions

$$\begin{aligned}
\mathbf{R}_A^a &= \nabla \times \mathbf{N}_G^a + \partial_0 \mathbf{N}_A^a - \mathbf{N}_E^a, \\
\mathbf{R}_G^a &= -\nabla \times \mathbf{N}_A^a + \partial_0 \mathbf{N}_G^a - \mathbf{N}_B^a.
\end{aligned} \tag{113}$$

In $\mathbf{R}_A^a$, $\mathbf{R}_G^a$ or $\mathbf{N}_E^a$, $\mathbf{N}_B^a$, respectively, we consider the currents as the leading terms in lowest-order perturbation theory, whereas all other terms are responsible for boson-boson interactions which are generally assumed to have a minor influence on the processes under consideration. Thus with respect to the lowest-order perturbation theory (111) and (112) can be written in the form

$$\frac{\partial^2}{\partial t^2} \tilde{\mathbf{A}}^a + \nabla \times \nabla \times \tilde{\mathbf{A}}^a + m_A^a \tilde{\mathbf{A}}^a = g_\chi \hat{\mathbf{j}}^a, \tag{114}$$

$$\frac{\partial^2}{\partial t^2} \tilde{\mathbf{G}}^a + \nabla \times \nabla \times \tilde{\mathbf{G}}^a - m_G^a \tilde{\mathbf{G}}^a = -\mathrm{i} g_\pi \hat{\mathbf{J}}^a. \tag{115}$$

These equations can be solved by means of Green functions, which leads to

$$\begin{aligned}
\tilde{A}_k^a &= \int \mathrm{d}^4 x' \mathcal{G}_A^a (x - x')_{kk'} g_\chi \hat{j}_{k'}^a (x'), \\
\tilde{G}_k^a &= \int \mathrm{d}^4 x' \mathcal{G}_G^a (x - x')_{kk'} (-\mathrm{i}) g_\pi \hat{J}_{k'}^a (x').
\end{aligned} \tag{116}$$

On the other hand by (102) the following current definition is suggested:

$$\tilde{j}_k^a = t_{aa'}^T j^{a'} k = \bar{\psi} t_{aa'}^T \sigma^{a'} \gamma_k \psi = \bar{\psi} \tilde{\sigma}^{a'} \gamma_k \psi. \tag{117}$$

These currents and the associated $\tilde{\sigma}$ and $\hat{\sigma}$ matrices have the explicit form

$$\begin{aligned}
\tilde{j}_k^1 &:= \bar{\psi} 2^{-1/2} (\sigma^1 - \mathrm{i}\sigma^2) \gamma_k \psi =: \hat{j}_k^2 \\
&= \text{charged current}, \\
\tilde{j}_k^2 &:= \bar{\psi} 2^{-1/2} (\sigma^1 + \mathrm{i}\sigma^2) \gamma_k \psi =: \hat{j}_k^1 \\
&= \text{charged current}, \\
\tilde{j}_k^3 &:= \bar{\psi} 2^{-1/2} (\sigma^3 + \sigma^0) \gamma_k \psi =: \hat{j}_k^3 \\
&= \text{neutral current}, \\
\tilde{j}_k^0 &:= \bar{\psi} 2^{-1/2} (\sigma^3 - \sigma^0) \gamma_k \psi =: \hat{j}_k^0 \\
&= \text{electromagnetic current}.
\end{aligned} \tag{118}$$

In an analogous way one obtains the explicit expressions for the magnetic currents $\tilde{J}_k^a$. The different current definitions (100) and (117) are enforced by the formalism, but correspond to the phenomenology. For instance the combination of states in neutron decay matrix elements in [25], (4.2.26) coincides with the combination $(\tilde{\sigma}^1 \hat{\sigma}^1)$ in the S-matrix element (126).

According to the definition of $\tilde{\sigma}^a$, (102) can be rewritten in the form

$$\Big[-\mathrm{i}\gamma^\mu(\partial_\mu - \mathrm{i}\frac{1}{2}g\tilde{A}^a_\mu\tilde{\sigma}^a - \frac{1}{2}\gamma^5\tilde{G}^a_\mu\tilde{\sigma}^a) + m\Big]\psi = 0. \quad (119)$$

If necessary one can introduce the temporal gauge for the vector fields in (119). In its magnetic part this equation is a generalization of Lochak's massless magnetic monopol equation (1.9) in [21], to magnetic electroweak interactions, and in addition this equation contains the full "electric" and "magnetic" electroweak interaction with corresponding massive fermions. For the sake of brevity the effect of symmetry breaking on the fermion masses is not treated in detail.

The Lagrangian density associated with this equation is defined by

$$\mathcal{L}_f(x) := \bar{\psi}\left[-\mathrm{i}\gamma^\mu\left(\partial_\mu + \mathrm{i}\frac{1}{2}g\tilde{A}^a_\mu\tilde{\sigma}^a + \frac{1}{2}\gamma^5\tilde{G}^a_\mu\tilde{\sigma}^a\right) + m\right]\psi. \quad (120)$$

Note that $\mathcal{L}_f$ vanishes as a consequence of the equation of motion. This means that the stationary value of the action integral is reached for $\mathcal{L}_f = 0$ (cf. [22], p. 58), and does not prevent its use.

The Lagrangian density (120) can be expressed by the "electric" and "magnetic" currents in the form

$$\mathcal{L}_f(x) = \bar{\psi}(-\mathrm{i}\gamma^\mu\partial_\mu + m)\psi + \frac{1}{2}g\tilde{A}^a_\mu\tilde{j}^\mu_a + \mathrm{i}\frac{1}{2}g\tilde{G}^a_\mu\tilde{J}^\mu_a. \quad (121)$$

Furthermore, in accordance with the foregoing remarks we add the "electric" and "magnetic" quark currents $\tilde{h}^a_\mu$ and $\tilde{H}^a_\mu$ to the leptonic currents on the right hand side of (114) and (115). In consequence these quark currents have to be added to the leptonic currents in (116). By substitution of these improved field representations in (121) one obtains in the case of temporal gauge

$$\begin{aligned}\mathcal{L}_f(x) &= \bar{\psi}(-\mathrm{i}\gamma^\mu\partial_\mu + m)\psi \\ &+ \frac{1}{2}g\tilde{j}^k_a(x)\int \mathrm{d}^4x'\mathcal{G}^a_A(x-x')_{kk'}[g_\chi\hat{j}^a_{k'}(x') + g_q\hat{h}^a_{k'}(x')] \\ &+ \frac{1}{2}\mathrm{i}g\tilde{J}^k_a(x)\int \mathrm{d}^4x'\mathcal{G}^a_G(x-x')_{kk'}[g_\pi\hat{J}^a_{k'}(x') + g'_q\hat{H}^a_{k'}(x')].\end{aligned} \quad (122)$$

Then for lepton-lepton interactions the "electric" part of the interaction Lagrangian density reads

$$\mathcal{L}^{\mathrm{ll}}_f(x) = \frac{1}{2}g\tilde{j}^k_a(x)\int \mathrm{d}^4x'\mathcal{G}^a_A(x-x')_{kk'}g_\chi\hat{j}^a_{k'}(x'), \quad (123)$$

while for lepton-quark interactions it is given by

$$\mathcal{L}^{\mathrm{lq}}_f(x) = \frac{1}{2}g\tilde{j}^k_a(x)\int \mathrm{d}^4x'\mathcal{G}^a_A(x-x')_{kk'}g_q\hat{h}^a_{k'}(x'). \quad (124)$$

The absence of $\mathcal{L}^{\mathrm{qq}}_f$ results from the absence of the quark Dirac equations which we did not introduce for brevity, but which can be derived in a complete lepton-quark dynamics (cf. [13], chapter 7).

Equation (119) can be treated by perturbation theory. Within the quantum mechanical formalism it can be shown that the S-matrix which corresponds to the first-order perturbation theory is given by the integral over the interaction Lagrangian density (cf. [23], (3.42), (3.53)). For the processes under consideration the corresponding S-matrix element reads

$$\begin{aligned}S &= \int \mathrm{d}^4x\mathcal{L}^{\mathrm{lq}}_f(x) \\ &= \frac{1}{2}gg_q\int \mathrm{d}^4x\mathrm{d}^4x'\tilde{j}^k_a(x)\mathcal{G}^a_A(x-x')_{kk'}\hat{h}^a_{k'}(x').\end{aligned} \quad (125)$$

According to its derivation, in this formula the currents have to be represented by the *free* lepton and quark states of their corresponding *free* Dirac equations. This leads to the following expression for (125) (cf. (3.60) in [23]):

$$\begin{aligned}S = \delta^4(P_\mathrm{f} - P_\mathrm{i} + p_\mathrm{f} - p_\mathrm{i})\Big(\frac{m^2}{E_\mathrm{f}E_i}\Big)^{1/2}\Big(\frac{M^2}{E^\mathrm{q}_\mathrm{f}E^\mathrm{q}_\mathrm{i}}\Big)^{1/2} \\ [\overline{u}(p_\mathrm{f}s_\mathrm{f})\tilde{\sigma}^a\gamma_k u(p_\mathrm{i}s_\mathrm{i})]\mathcal{G}^a_A(p_\mathrm{f}-p_\mathrm{i})_{kk'}[\overline{u}(P_\mathrm{f}s_\mathrm{f})\hat{\sigma}^a\gamma^{k'}u(P_\mathrm{i}s_\mathrm{i})],\end{aligned} \quad (126)$$

where $\mathcal{G}^a_A(k)$ is the four-dimensional Fourier transform of the Green function, while the capital letters denote the quark momenta of initial and final states, and the small letters denote the leptonic initial and final states.

The Green function and its Fourier transform can be exactly calculated (cf. (12.119) in [24]), and one obtains

$$\mathcal{G}^a_A(\omega,\mathbf{k})_{kk'} = [\mathbf{1} + \frac{1}{(m^{a2}_A - \omega^2)}\mathbf{k}\otimes\mathbf{k}]\frac{1}{(\mathbf{k}^2 + \omega^2 - m^{a2}_A)}. \quad (127)$$

If one compares the S-matrix element (126) with that of the phenomenological theory, for instance (4.2.3) in [25], one obviously realizes a general agreement except for three pecularities:

(i) the absence of helicity projection operators;

(ii) the absence of the Cabbibo angle in the quark mass matrix;

(iii) the altered boson mass matrix in comparison with the phenomenological one.

As far as the helicity operators are concerned, their absence has no relation to the subject under consideration. In the introduction it was pointed out that massive neutrinos imply the use of an $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ theory ([26], chapter 6), which on the microscopic level can be introduced without difficulties. Only for economic reasons the theory has been based on the less complicated $SU(2) \otimes U(1)$ version in this paper.

In addition, concerning (ii) the appearance of the Cabbibo angle depends on the calculation of a nontrivial quark mass matrix which is not the topic treated here, but which was already calculated in the spinor theory [27].

On the other hand the alteration of the boson mass matrix in comparison with the phenomenological one is the crucial result of this treatment, because it is an immediate consequence of the combined CP- and isospin symmetry breaking. According to (106) the electric boson mass matrix reads explicitly

$$m_A^a \delta_{aa'} = \begin{pmatrix} \mu_A^2 - a_A & 0 & 0 & 0 \\ 0 & \mu_A^2 + a_A & 0 & 0 \\ 0 & 0 & \mu_A^2 + a_A & 0 \\ 0 & 0 & 0 & \mu_A^2 - a_A \end{pmatrix}, \tag{128}$$

where the last row represents the photon mass.

If this photon mass is to vanish, then from (128) it follows that also the mass of the $\tilde{A}^1$-vector boson must vanish, while the mass of the $\tilde{A}^2$-vector boson is $2\mu^2$. Even if for a more refined Weinberg transformation the consequences for the $\tilde{A}^1$- and $\tilde{A}^2$-masses are not so drastic as in the present case, the essential result of our calculation can be summarized by the following theorem:

Theorem 3: The simultaneous breaking of CP- and isospin symmetry on the parton level implies a correlation between the masses of the charged and the neutral electroweak vector bosons in the associated effective theory.

6. Conclusions

Studies of electroweak nuclear reaction rates show that these undergo considerable variations in dependence on the individual nuclear and electronic structure of the atoms involved [28]. Therefore it is not possible to derive universally valid predictions about the results of experiments with electroweak reactions, as the corresponding calculations must be done with inclusion of the special nuclear and electronic structure and the related energetic and phase space constraints.

On the other hand one can look for properties which in any case are relevant for such processes and which represent a prerequisit that such processes proceed at all. Obviously under this category fall the modifications of the basic electroweak laws which are caused by symmetry breaking. The foregoing investigations are concerned with this problem. They convey a qualitative insight into elementary processes which govern these modifications in general, but no statements are given which are referred to special matter configurations.

For the corresponding calculations a special model is used. In this model it is assumed that electroweak bosons, leptons and quarks possess a substructure of elementary fermionic constituents. In the case of CP- and isospin symmetry breaking an analysis of elementary nuclear electroweak reaction rates leads to considerable changes of these rates compared with the symmetry conserving theory. In essence these changes have two sources:

(i) The charged sector of the vector bosons undergoes a synchronous mass splitting with the neutral sector. This mass splitting leads to very light negatively charged bosons and very heavy positively ones. As the bare coupling constants of the electromagnetic and of the weak processes are of the same magnitude (cf. (10.41), (10.42) in [29]), and the same holds for the masses of the W^--boson and the photon, the transition rates for these bosons must be of the same magnitude too. In consequence this produces a considerable change of the transition rates of processes where the W^--bosons are involved in comparison with the symmetry conserving case. On the other hand due to the increase in the mass of the W^+-boson corresponding processes are suppressed.

(ii) The charged fermions of the CP-symmetric theory are transmuted into dyons if the CP-symmetry is broken, a fact which is independent of isospin symmetry breaking. As the electroweak electronic capture and the bound state β-decay depend on the electronic wave functions, the transmutation to dyons is expected to induce via the additional magnetic fields considerable changes in these wave functions, which in any case will influence the corresponding reaction rates.

Can these results be related to realistic experiments? At least some qualitative agreements can be established. The theory leads to an extension of the standard model physics. This coincides with the assumption of Urutskoev et al. that their experiments must be connected with a violation of the conventional electroweak reaction schemes, possibly triggered by light magnetic monopoles [10, 11, 30 – 32]. In particular with respect to magnetic interactions, the theory, here developed, predicts the existence of additional magnetic bosons, i. e. additional magnetic forces and dyons which points in the same direction as the ideas of Urutskoev and Lochak.

But the experiments of Urutskoev et al. are rather intricate. Discharges between metallic foils in vessels filled with various fluids lead to the evidence of numerous elements being not present in the system before the explosion and depending on the special foils and fluids. In spite of a lot of semiempirical studies, see for instance [11, 12], the physical mechanism producing these results is unknown. Only the action of strong forces is definitely excluded.

Thus, before one can give any quantitative account of these observations, an appropriate mechanism must be found which underlies these phenomena. This includes an explanation why just these experiments and no or any other ones are able to enforce a deviation from conventional standard model physics. Therefore theoretically the most urgent problem is to show in which way the theoretical CP-symmetry breaking mechanism can be related to the experimental arrangements described above. In view of the complexity of the theoretical as well as of the experimental investigations this is not an easy task and will be treated and continued elsewhere. Finally it should be noted that the experiments of Urutskoev et al. are not the only attempts in that direction of research, they are only the best documented ones, mainly by numerous articles in Russian journals. For similar research in USA with laser action in cavities, see [33].

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